MIT Dept. of Mathematics
Benjamin Seibold

18.086 spring 2008
Exercise Sheet 3

Out Fri 03/07/08 Due Fri 03/21/08

Exercise 7 Consider the Korteweg–de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0$$

1. Using your knowledge about Airy's equation $u_t + u_{xxx} = 0$, explain how solutions to the KdV equation are expected to differ from solution to Burgers' equation $u_t + 6uu_x = 0$.
2. Write a Matlab program that approximates the KdV equation by finite differences. Consider the interval $[-1, 1]$ with periodic boundary conditions. I suggest an explicit step for the nonlinear advection and an implicit step for the dispersion term, but you are free to use other methods. In order to get a reasonable resolution you should use at least 300 grid points.
3. Define the function
$$f_c(x) = \frac{c}{2}\mathrm{sech}^2\left(\frac{\sqrt{c}}{2}x\right)$$
where $\mathrm{sech}(x) = \frac{2}{e^x + e^{-x}}$. You can verify as a private exercise that for every $c > 0$ indeed $u(x,t) = f_c(x - ct)$ is a solution to the KdV equation (such travelling waves are called *solitons*). Run your program with the following initial data
   (a) $u_0(x) = f_{400}(x)$
   (b) $u_0(x) = f_{400}(x + 0.7) + f_{200}(x)$
   (c) $u_0(x) = \frac{1}{2}\left(f_{400}(x + 0.7) + f_{200}(x)\right)$

   Plot the results at time $t = 0.015$. Explain briefly how the three cases behave. In particular explain how the nonlinearity becomes visible in case (b).

Exercise 8 Download the matlab file `mit18086_levelset_front.m` from the CSE web page, and run it with `mit18086_levelset_front(1)` (a fire front) and `mit18086_levelset_front(2)` (movement under curvature).

1. Investigate the role of the reinitialization by comparing the cases of zero, one, and five reinitialization steps per time step. Explain what too much reinitialization does to the geometry?
2. In the fire front case, is the correct area of burned ground connected at $t = 2$?

In the movement under curvature case, how many connected components does the correct geometry have at $t = 2$? For both cases, provide numerical results that show the correct topologies.