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Exercise Sheet 2

Out Wed 02/22/08 Due Fri 03/07/08

Exercise 4
Look at the matlab file `mit18086_error_analysis.m` on the course web page.

1. Run the program with the three different initial conditions provided in the file. Explain the results.
2. Modify the Lax-Wendroff method by multiplying the added viscosity by a factor of 1.01. Compare with the original Lax-Wendroff scheme and explain the result.
3. Include the Crank-Nicolson method for the transport equation, as given in the file `mit18086_fd_transport_growth.m` and compare to the other methods.

Exercise 5
Consider the convection-diffusion equation $u_t = u_x + du_{xx}$. Implement the three candidate methods given in section 6.5 in the book *Computational Science and Engineering*, i.e.

1. forward in time, centered convection, centered explicit diffusion
2. forward in time, upwind convection, centered explicit diffusion
3. explicit convection, implicit diffusion

and test them, firstly for a large, secondly for a medium, and thirdly for a very small diffusion constant d. Explain your results.

Exercise 6
Consider Burgers' equation

$$u_t + uu_x = 0$$

with a Gaußian initial condition $u_0(x) = \exp(-x^2)$. Choose your computational interval large enough, such that the solution is essentially 0 at the boundaries.

Compare the numerical solution obtained by a nonconservative upwind method to the solution obtained by the conservative upwind method

1. up to a time at which the analytical solution is still smooth
2. up to about twice the time after a shock has appeared in the analytical solution.