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Modeling Elastic Wave Propagation in layered medium with Surface Topography

(Term Report for CSE-2: 18.086 - 2008)

0. Abstract

The objective of the Term Project is to study the Finite Difference Methods to better approximate and model the effect of topography on Wave Propagation. First, wave propagation is modeled using Standard Staggered grid in a 2D homogenous medium (in a 1000m*500m space) bounded by rigid surfaces on three sides and a free surface on the fourth side. Then, PML boundaries conditions are introduced to suppress the waves from rigid surfaces. Vacuum method and Image method for Surface topography are discussed. Both are implemented for horizontal plane surface and compared. Vacuum method is chosen for modeling Surface Topography. A dipping layer is introduced into the model now. To take, a step further, multi-grid/ variable grid is developed and implemented for plane surface case.

1. Introduction

Understanding wave propagation is particularly important for processing the seismic signal recorded during Seismic Survey for Oil Exploration. With conventional oil reserves depleting, we are now forced to explore oil in extreme terrains as well. Due to the uneven topography most of the energy sent into the ground turns out on seismograms as scatter noise. It is, therefore, natural to understand the wave propagation in such topography for us to remove the noise due to scattering. We do are interested in exploring the possibilities of using these surface scatters as secondary sources to better image the reservoir and enhance resolution. .

Conventionally, finite differences are used to model the wave propagation. But, approximating the topography is a problem due to the complexity in approximating the topography with a Cartesian grid and the resolution required. It is also noticed that the corners of the grids would act as diffractors and disturb the solution. These aspects are observed in the project. .

I have applied the model to the Seismic Survey done in a field trip to compare how well we could do to simulate the wave propagation. The Seismic Survey is done in the California Desert at Vidal (River Side Mountains). The geological structure to be imaged was the fault below. .

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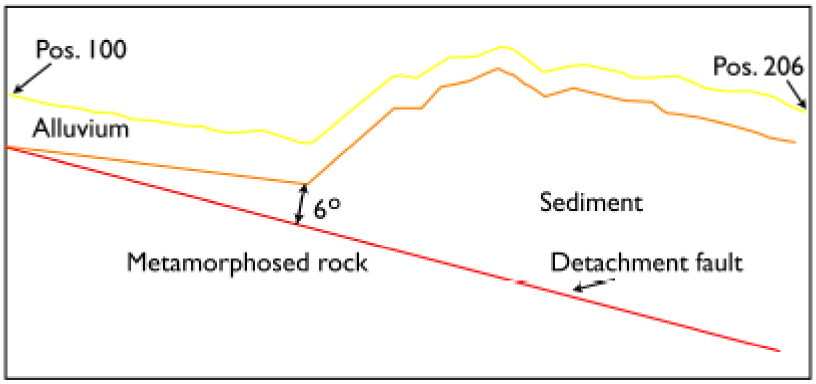
A schematic diagram of the feature is shown below.



We have deployed Geophones along the surface (at every 30m for 1 kilometer) at equal intervals and shot Betsy gun at certain source locations (at every 30m along the seismic line) to record the seismogram. Analyzing the seismograms by refraction seismology we have arrived at the above model. I would like to verify if our model would reproduce the observed seismograms.

I would like to present the physics behind wave propagation and then proceed to the modeling and results.

2. Physics of Elastic Wave Propagation .

Consider a Homogenous, Isotropic and 3-D infinite medium. Let the material properties be given by: E – Young's Modulus, K- Bulk Modulus, G- Shear Modulus, ν – Poisson's ratio. But, all the above four parameters are inter related and can be expressed in terms of only two parameters. Usually Lame's constants (λ, μ) are used to denote the medium parameters. These are related to the above parameters as: .

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}; \qquad \mathrm{K} = \lambda + \frac{2\mu}{3}; \qquad \mathrm{G} = \mu; \qquad \nu = \frac{\lambda}{2(\lambda + \mu)};$$

Consider the infinitesimal cube presented below. σ_{xx}, σ_{yy}, σ_{zz} are the Normal Stresses acting on the faces of the element **and** $\sigma_{xy}, \sigma_{yx}, \sigma_{zx}, \sigma_{xz}, \sigma_{yz}, \sigma_{zy}$ are the shear stresses acting on the faces of the cube. The strains are given by: .

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$$\varepsilon_{xx} = \left(\frac{\partial u}{\partial x}\right); \quad \varepsilon_{yy} = \left(\frac{\partial v}{\partial y}\right); \quad \varepsilon_{zz} = \left(\frac{\partial w}{\partial z}\right); \quad \varepsilon_{xy} = \left(\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right) \quad \varepsilon_{yz} = \left(\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right);$$

$\varepsilon_{zx} = \left(\frac{\partial u}{\partial z}\right) + \left(\frac{\partial w}{\partial x}\right)$ where u,v,w are displacements along +ve x,y,z directions.

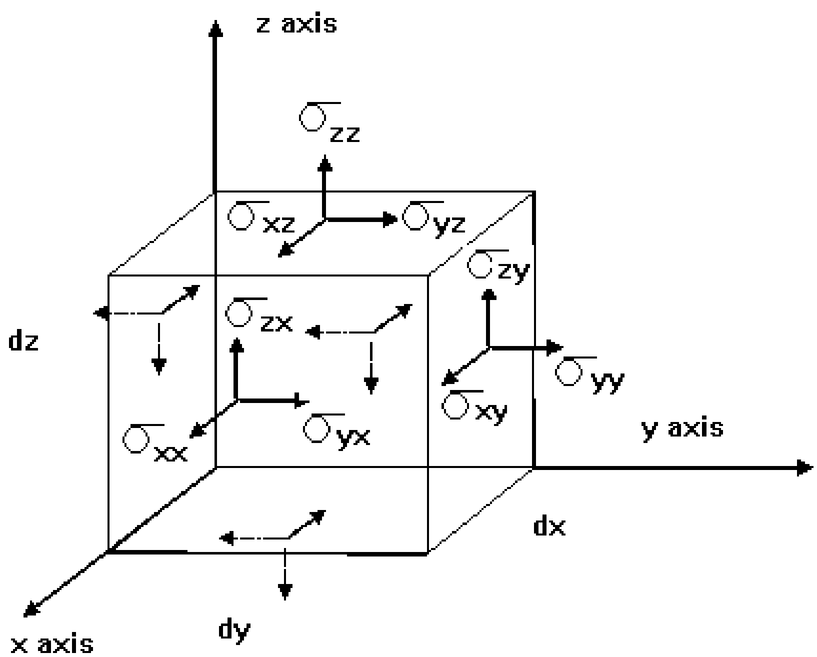

Figure showing the directions of stress components. .

Stress and strain are related as: .

$$\sigma_{xx} = \lambda\bar{\varepsilon} + 2\mu\varepsilon_{xx}; \; \sigma_{yy} = \lambda\bar{\varepsilon} + 2\mu\varepsilon_{yy}; \; \sigma_{zz} = \lambda\bar{\varepsilon} + 2\mu\varepsilon_{zz}; \; \sigma_{xy} = \mu\varepsilon_{xy}; \; \sigma_{yz} = \mu\varepsilon_{yz}; \; \sigma_{zx} = \mu\varepsilon_{zx}$$

Applying equation of motion to the above cube we arrive at :

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}; \quad \rho\frac{\partial^2 v}{\partial t^2} = \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}; \quad \rho\frac{\partial^2 w}{\partial t^2} = \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}$$

The above differential equations describe the wave motion. For simplicity of modeling we consider a 2D model. So our equations reduce to: .

$$\sigma_{xx} = (\lambda + 2\mu)\left(\frac{\partial u}{\partial x}\right) + \lambda\left(\frac{\partial v}{\partial y}\right)$$

$$\sigma_{yy} = (\lambda + 2\mu)\left(\frac{\partial v}{\partial y}\right) + \lambda\left(\frac{\partial u}{\partial x}\right)$$

$$\sigma_{xy} = \mu\left(\left(\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right)\right)$$

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \quad \text{and} \quad \rho\frac{\partial^2 v}{\partial t^2} = \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}$$

We can rewrite the above equations as below, known as **Velocity – Stress Formulation.** Since, we would be recording velocities in field and boundary conditions are associated with stresses, it is a better formulation.

$$\dot{\sigma}_{xx} = (\lambda + 2\mu)\left(\frac{\partial V_x}{\partial x}\right) + \lambda\left(\frac{\partial V_y}{\partial y}\right)$$ ‘ . ’ indicates derivative w.r.t time ---------------- 1

$$\dot{\sigma}_{yy} = (\lambda + 2\mu)\left(\frac{\partial V_y}{\partial y}\right) + \lambda\left(\frac{\partial V_x}{\partial x}\right)$$ ---------------- 2

$$\dot{\sigma}_{xy} = \mu\left(\left(\frac{\partial V_y}{\partial x}\right) + \left(\frac{\partial V_x}{\partial y}\right)\right)$$ ---------------- 3

$$\rho\frac{\partial V_x}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}$$ ---------------- 4

$$\rho\frac{\partial V_y}{\partial t} = \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y}$$ ---------------- 5

We would use finite difference scheme to solve the above system of differential equations

3. Standard Staggered Grid

The two popular methods of modeling the elastic wave propagation are Standard Staggered Grid (J. Virieux et. Al, 1986) and Rotated Staggered Grid (Saenger et.al, 1997). Standard Staggered grid is the traditional one. Though, rotated staggered is good at handling material discontinuity and easier to code and implement, I have chosen Standard Staggered Grid because the surface condition (Normal Stress is zero) can be implemented well by image method. And I see smart imaging of stresses as the best way for Topography. Whereas, rotated staggered method is not so comfortable in implementing the stress imaging methods, due to rotation in the co-ordinate axes.

Standard Staggered Grid is as below. Consider a cell as shown. Axial Stresses and Elastic modules (σ_{xx}, σ_{yy},λ,μ) lie at the centre of the cell, shear stress and density (σ_{xy},ρ) at the corners and the velocities

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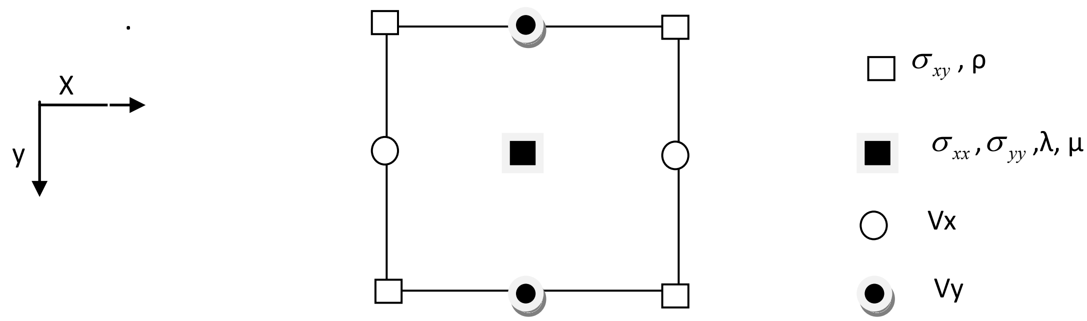
on the edges. The velocities in x and y direction are staggered by half a grid distance in both directions. Stresses lie in between the velocities and as a result they are also staggered. Also, the velocities and stresses are not coexistent in time. They are staggered by half time step.



Implementing this grid to our differential equations, our finite differences equations look like:

$$\frac{\left(\sigma^{m+0.5}{}_{xx(i,j)} - \sigma^{m-0.5}{}_{xx(i,j)}\right)}{\Delta t} = (\lambda + 2\mu)\left(\frac{\left(V^{m}{}_{x(i,j+0.5)} - V^{m}{}_{x(i,j-0.5)}\right)}{\Delta x}\right) + \lambda\left(\frac{\left(V^{m}{}_{y(i+0.5,j)} - V^{m}{}_{y(i-0.5,j)}\right)}{\Delta y}\right)$$

$$\frac{\left(\sigma^{m+0.5}{}_{yy(i,j)} - \sigma^{m-0.5}{}_{yy(i,j)}\right)}{\Delta t} = (\lambda + 2\mu)\left(\frac{\left(V^{m}{}_{y(i+0.5,j)} - V^{m}{}_{y(i-0.5,j)}\right)}{\Delta y}\right) + \lambda\left(\frac{\left(V^{m}{}_{x(i,j+0.5)} - V^{m}{}_{x(i,j-0.5)}\right)}{\Delta x}\right)$$

$$\frac{\left(\sigma^{m+0.5}{}_{xy(i,j)} - \sigma^{m-0.5}{}_{xy(i,j)}\right)}{\Delta t} = \mu\left(\left(\frac{\left(V^{m}{}_{y(i,j+0.5)} - V^{m}{}_{y(i,j-0.5)}\right)}{\Delta x}\right) + \left(\frac{\left(V^{m}{}_{x(i+0.5,j)} - V^{m}{}_{x(i-0.5,j)}\right)}{\Delta y}\right)\right)$$

$$\rho\frac{\left(V^{m+1}{}_{x(i,j)} - V^{m-1}{}_{x(i,j)}\right)}{\Delta t} = \left(\frac{\left(\sigma^{m+0.5}{}_{xx(i,j+0.5)} - \sigma^{m+0.5}{}_{yy(i,j-0.5)}\right)}{\Delta x}\right) + \left(\frac{\left(\sigma^{m+0.5}{}_{xy(i+0.5,j)} - \sigma^{m+0.5}{}_{xy(i-0.5,j)}\right)}{\Delta y}\right)$$

$$\rho\frac{\left(V^{m+1}{}_{y(i,j)} - V^{m-1}{}_{y(i,j)}\right)}{\Delta t} = \left(\frac{\left(\sigma^{m+0.5}{}_{yy(i+0.5,j)} - \sigma^{m+0.5}{}_{yy(i-0.5,j)}\right)}{\Delta y}\right) + \left(\frac{\left(\sigma^{m+0.5}{}_{xy(i+0.5,j)} - \sigma^{m+0.5}{}_{xy(i-0.5,j)}\right)}{\Delta x}\right)$$

Note: ‘j’ is along x-axis and ‘I’ is along y axis. .

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Stability and Accuracy of the Method:

After analyzing the finite difference equations for stability by Von Neumann Analysis we arrive at the following condition:

$$\Delta t \leq \frac{h}{c_I}$$

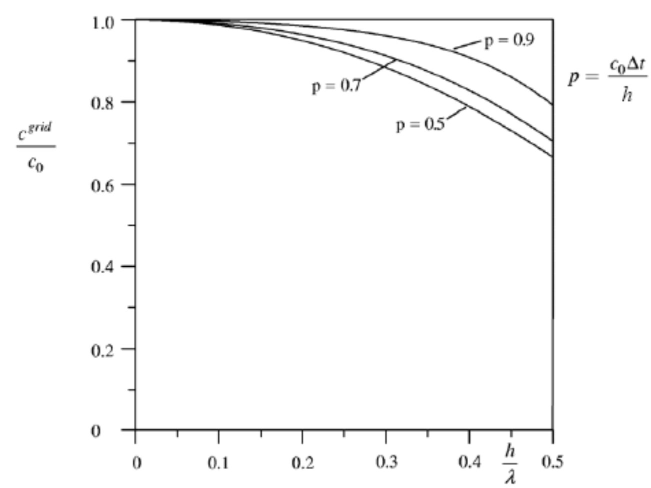
Where, c_I is the Velocity of the wave at any grid point. So, we take it to be the Maximum Velocity possible in the domain. This very intuitive result which says that the grid velocity ($\frac{\Delta h}{\Delta t}$) should be at least equal to the Wave Velocity.From Taylors expansion, we can see that the method is Second Degree accurate in space and Time as well. (Since it is central Scheme in space and Time)

Grid Dispersion:

Though the method is stable if it satisfies the above mentioned condition, we may end up with inaccurate solutions if we don't have enough grid resolution for the wavelengths we are interested in. We should sample at least 10 grid points per wavelength. This result comes from the dispersion analysis. We look at the ratio of grid velocity (which is the group velocity of waves, cgrid) to phase velocity (which differs for different frequencies if the grid velocity is not close enough to wave velocity in the media).

$$c^{grid} = \frac{\omega}{k} = \frac{h}{\pi \Delta t} \frac{\lambda}{h} \arcsin\left(c_0 \frac{\Delta t}{h} \sin \frac{\pi h}{\lambda}\right) \qquad v_g^{grid} = \frac{\partial \omega}{\partial k} = \frac{c_0 \cos \frac{\pi h}{\lambda}}{\left[1 - \left(c_0 \frac{\Delta t}{h} \sin \frac{\pi h}{\lambda}\right)^2\right]^{1/2}} .$$

See that both are same at wave length = 10*h;



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Boundary Conditions and Material Interfaces:

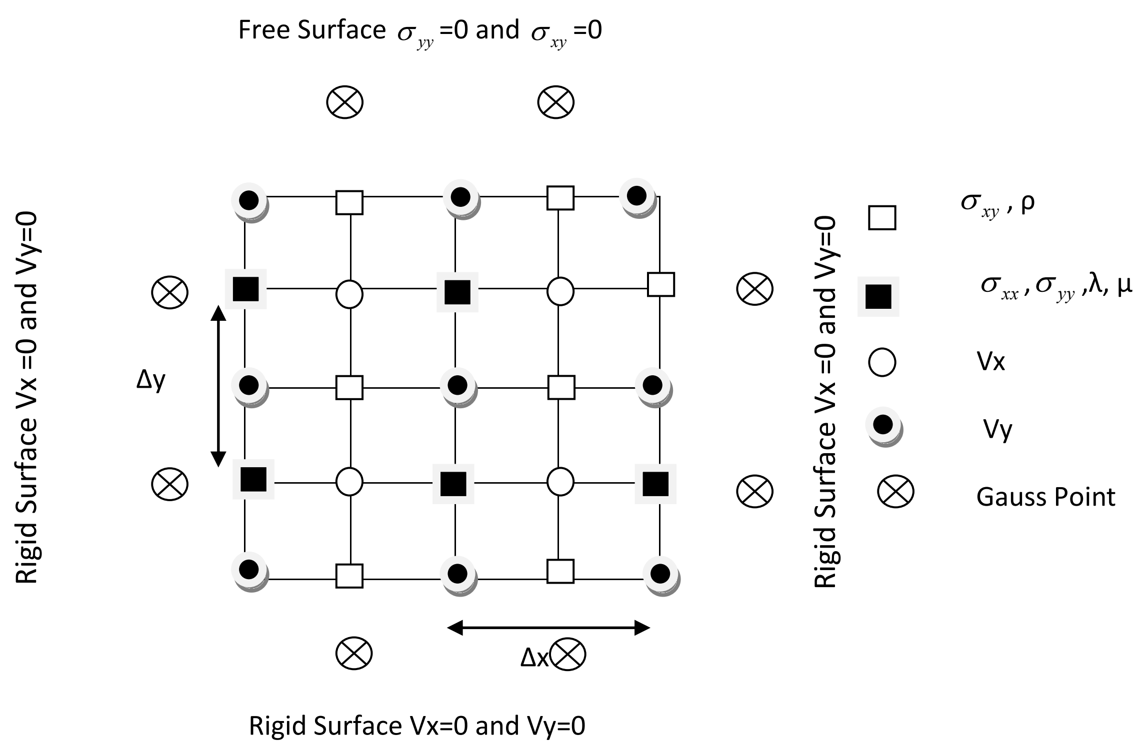
Fixed Boundary:

Fixed boundary is simulated by applying Velocities to be zero at the boundary. But, at the boundary, since we are using staggered grid, we don't have the both Vx and Vy simultaneously. So, we put whatever the Velocity point available to zero. The other is conceptually made zero by assuming a gauss point on the other side of the grid.

Free Boundary:

Fixed boundary is simulated by applying Normal Stress and Shear Stress to be zero (σ_{yy} =0 and σ_{xy} =0) at the boundary. But, at the boundary, since we are using staggered grid, we don't have both σ_{yy} and σ_{xy} simultaneously. So, we put whatever the Stress point available to zero. The other is conceptually made zero by assuming a gauss point on the other side of the grid.

Gauss points are used to satisfy the boundary conditions as shown in the figure.



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Material Interfaces:

The interaction at material interfaces is automatically taken care (as all the Stresses and Displacements are Continuous) by using the harmonic average of Elastic Parameters at Grid points around and Arithmetic average of densities for the points on the material interfaces.

PML Boundary Conditions:

Since, I want to use a source at surface and want to record only reflections from layers below and reflections due to material discontinuity, I don't want a perfect rigid conditions at all the other three sides. So, I want the wave to be damped at the boundaries, except the surface.

This is accomplished by implementing Perfectly Matching Layer Boundary Conditions. Here, we transform the x and y axes to complex plane given by the transformation below:

$x \rightarrow x + i\,\frac{d(x)}{\omega}$ and $y \rightarrow y + i\,\frac{d(y)}{\omega}$; $d(x) and\ d(y)$ are chosen such that they are equal to zero within the domain and a positive values depending on the damping you need, outside the domain. This implies that we solve the same differential equation within the domain. We add few more layers of grid to damp the wave outside the domain. So, now the boundary is just not a single line but a region where the energy is damped.

For implementing the PML conditions, transformations for x and y are not applied simultaneously. We decouple the system and solve once transforming x and the transforming y and then club them. Our differential equations:

$$
\begin{aligned}
\varrho\,\frac{\partial v_x}{\partial t} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \quad ; \quad \frac{\partial \sigma_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_y}{\partial y} \\
\varrho\,\frac{\partial v_y}{\partial t} &= \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \quad ; \quad \frac{\partial \sigma_{yy}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_y}{\partial y} + \lambda \frac{\partial v_x}{\partial x} \\
& \qquad\qquad\qquad\qquad\quad ; \quad \frac{\partial \sigma_{xy}}{\partial t} = \mu\frac{\partial v_y}{\partial x} + \mu \frac{\partial v_x}{\partial y}\ ,
\end{aligned}
$$

These would transform to:

$$
\begin{aligned}
& v = v^{\|} + v^{\perp} && ; \quad \sigma = \sigma^{\|} + \sigma^{\perp} \\
& \varrho\,(\frac{\partial}{\partial t} + d(x))v_x^{\perp} = \frac{\partial \sigma_{xx}}{\partial x} && ; \quad \varrho\,\frac{\partial v_x^{\|}}{\partial t} = \frac{\partial \sigma_{xy}}{\partial y} \\
& \varrho\,(\frac{\partial}{\partial t} + d(x))v_y^{\perp} = \frac{\partial \sigma_{xy}}{\partial x} && ; \quad \varrho\,\frac{\partial v_y^{\|}}{\partial t} = \frac{\partial \sigma_{yy}}{\partial y}
\end{aligned}
$$

$$
\begin{aligned}
&(\frac{\partial}{\partial t} + d(x))\sigma_{xx}^{\perp} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} \quad ; \quad \frac{\partial \sigma_{xx}^{||}}{\partial t} = \lambda \frac{\partial v_y}{\partial y} \\
&(\frac{\partial}{\partial t} + d(x))\sigma_{yy}^{\perp} = \lambda \frac{\partial v_x}{\partial x} \quad ; \quad \frac{\partial \sigma_{yy}^{||}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_y}{\partial y} \\
&(\frac{\partial}{\partial t} + d(x))\sigma_{xy}^{\perp} = \mu \frac{\partial v_y}{\partial x} \quad ; \quad \frac{\partial \sigma_{xy}^{||}}{\partial t} = \mu \frac{\partial v_x}{\partial y} \ .
\end{aligned}
$$

For detailed study one may refer to G. Festa. Et al

4. Surface Topography

As we have discussed earlier, we can handle material to material transformation easily. Then, why do we have problems with Free Surface? The reason is that we no longer have Stress Continuity and the contrast is so high. σ_{xx} is discontinuous at the boundary. We need to treat the Free Surface specially such that we satisfy the condition that normal and shear stresses are zero at the Free Surface. Since we can approximate surface only as discrete blocks, Finite Element Methods are more suitable for surface topography problems. But, the problem finally boils down to the computational expense we can afford. Finite element methods are more computationally expensive and are not suitable for 3D seismic imaging given the present capabilities of computers. So, it's more important to develop efficient methods in Finite Differences to better approximate the Free Surface Topography

Two methods well discussed in the literature are:
1) Direct Method/ Vacuum Method (Hestholm et.al, 1994) .
2) Imaging Method (Robertsson et. al 1996, 1997)

Direct Method/ Vacuum Method:

Since, we are using staggered grid, we don't have all the stresses on the boundary however we may choose to discretize the Surface. We end up having either shear stress or Normal Stress to be on the Surface. When we have complex surface both may appear on the surface, but not at the same space location. So, direct method relies on the fact that we apply zero normal stress and zero shear stress in two layers, but not one layer. So, our boundary is not distinct but diffuse. When we talk in terms of two layers to implement boundary it is referred to as Direct Method in the literature.

But, implementing the above conditions, though on two layers by picking up point to point along the surface when there is topography is difficult. So, people have come up with implementing the above conditions by assuming the velocity of air above the Free Surface is zero and Shear/ Normal Stress whichever is existing on the boundary is zero. By putting the Elastic coefficients of grid points above the user defined boundary to be zero, we accomplish the previously stated B.C.s in two layers at h and h/2.

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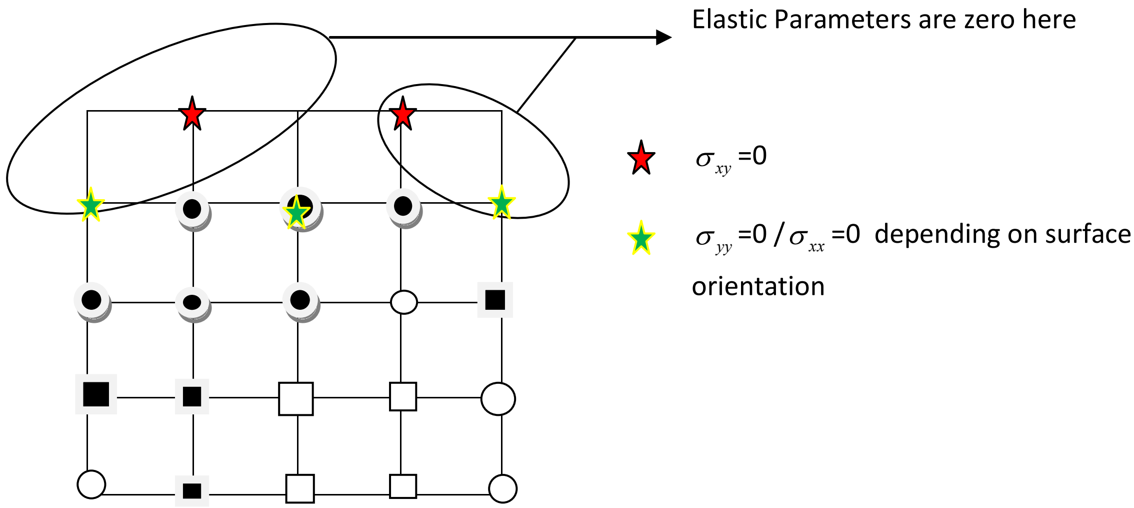
Where h is the grid spacing normal to the surface. This is like treating the medium above the surface as vacuum. The figure illustrates the method.

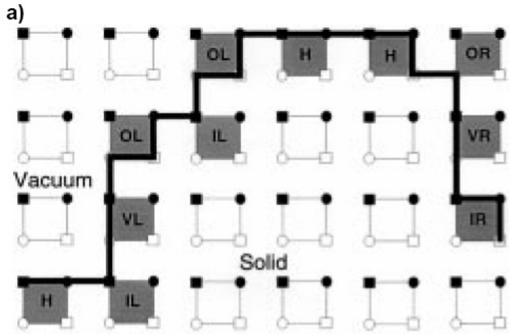


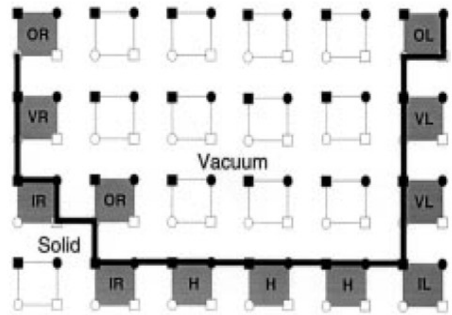
I have implemented Vacuum Method in my code for Surface Topography. We have compared both the methods for flat surface.

Image Method

Image Method is introduced by Robertsson et. Al (1996). This method implements stress free conditions along a single layer but not two. This is based on the Gauss Point idea we have discussed earlier. We force Normal Stress and Shear Stress to go to zero at the same space point by assuming a gauss point. Here, we won't assume Elastic parameters to go to zero in the grid points above the free surface. But, we take inverse of density to go to zero so that while applying the numerical scheme velocities automatically go to zero. We need not apply special attention to make velocities above free surface to go to zero.

Though it sounds so simple, the major problem with the method is how to image as the normal to the boundary changes and what about points in the corner. To address this problem Robertson has classified the entire possible outcome into 7 cases as shown below and addresses how to deal with each case. Its critical at corners. Except that, either we have to image in x direction or Y direction. Imaging the stresses along x and y axes sequentially will avoid the problem of cross imaging.





(Figures taken from Robertsson et. al). The Seven Possible Cases are shown here

Solid Circles Represent Vx Components, Light Circles the Vy Components, Light squares represent σ_{xy} Solid Squares represent σ_{yy} σ_{xx}

Separately for vertical and horizontal derivatives, the computational procedure can be summarized as follows:

1) Classify the free surface boundary points before FD calculations
2) Set reciprocal values of densities to zero above free surface
3) First Stress conditions are updated in every step. Normal Stresses along horizontal and vertical boundary are made to go to zero. Inner and outer corners have all stresses go to zero
4) Particle velocities are calculated by imaging along vertical direction only. Then by imaging along x direction and both are added

I couldn't implement this method right now because of the difficulty in formulating the imaging and classification algorithm in this short time available.

But, I have implemented the method for flat surface and compared it to the vacuum Method.

COMMENT: Both the methods discussed here are not completely accurate for we still discrete the surface as stair cases. This produces some diffraction effects at the corners.

5. Multi- Grid/ Non-Uniform grid

My ultimate goal is to model Surface Scattering. So, I would wish to discretise the boundary as best as possible and also need more resolution at the surface to study the scattering phenomenon. But, I cannot increase the resolution in the complete computational domain. So, I chose to solve the problem by refining the mesh near the surface. The idea is:

1) Solve the FD scheme for velocities on the entire grid
2) At the Fine-Coarse grid interface interpolate the velocities.
3) Using this as boundary for fine grid, solve the FD scheme for solving velocities on fine grid.

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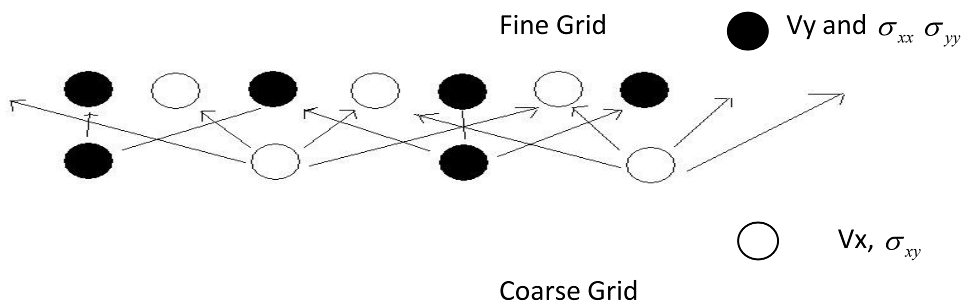
4) Update the velocities on Coarse grid in fine grid region by interpolating.
5) Now solve for Stresses on Coarse grid and then on fine grid. Repeat the process.

The interpolation scheme from Coarse to fine grid (along x direction) is as below. I have created interpolation matrices for the below in x and y directions and then restricted to find the coarse grid values from fine grid.



6. Modeling, Results and Discussions.

I would present the numerical experiments carried out in the following order and present discussions in the order.

1) Comparison between models run with PML boundary conditions and without PML conditions.
2) Effect of Source Frequency and Grid Dispersion effects.
3) Comparison between Vacuum Method and Image Method for flat Free Surface.
4) Implementation of Multi-Grid to Flat Free Surface with image method.
5) Surface Topography modeling for the field problem with and without fault.

For the first four cases the model is a 1000m long and 500 m deep 2D region with PML Boundary conditions on right, left and bottom boundaries. Free surface is on the Top.

Source: A Ricker wavelet of dominant frequency 25Hz is used for 1-4 and a source of 40 Hz is used for case 5. Source location is at 40^{th} grid point for the first four cases and on the 196^{th} grid point on the 5^{th} case. Source is always located on the Surface. We include source by adding a forcing term in the differential equation to calculate Vx. This force is given by Ricker wavelet.

Receivers: For the first four cases receivers are located at 30^{th} 50^{th} and 70^{th} grid point, while for 5^{th} case we have receivers at every 6^{th} grid point starting from 11. This is to simulate the field studies.

Grid Spacing and Time: Grid spacing is taken as 5m in x and y directions for all the cases except multigrid where fine grid have a special resolution of 2.5 m. For first 4 cases we have use 200*100 grid and for the 5^{th} case I have used 221*100 grid. Time steps of 0.5*10^-03 are used.

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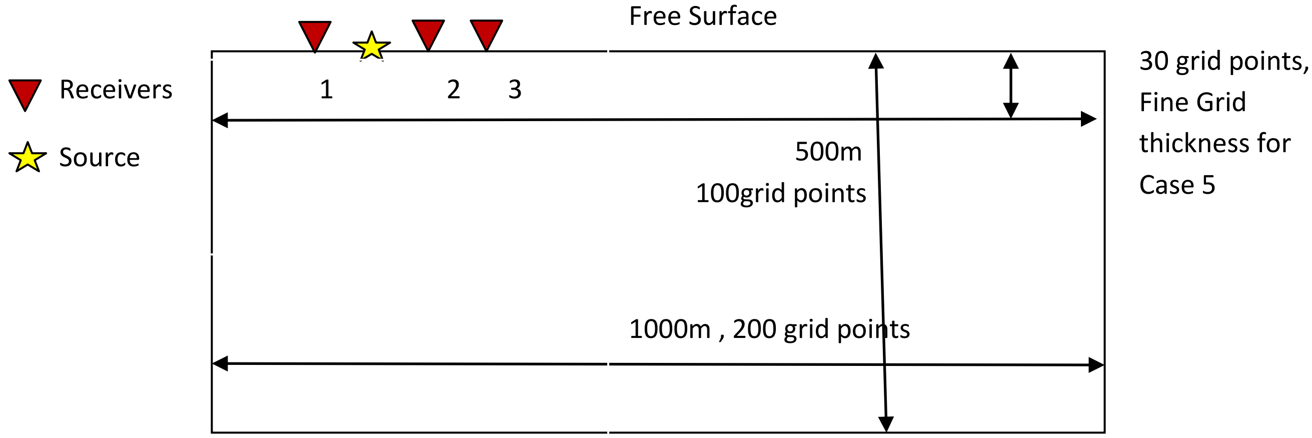
PML Boundary Layer: Where ever present PML layer is 10 grid points thick.

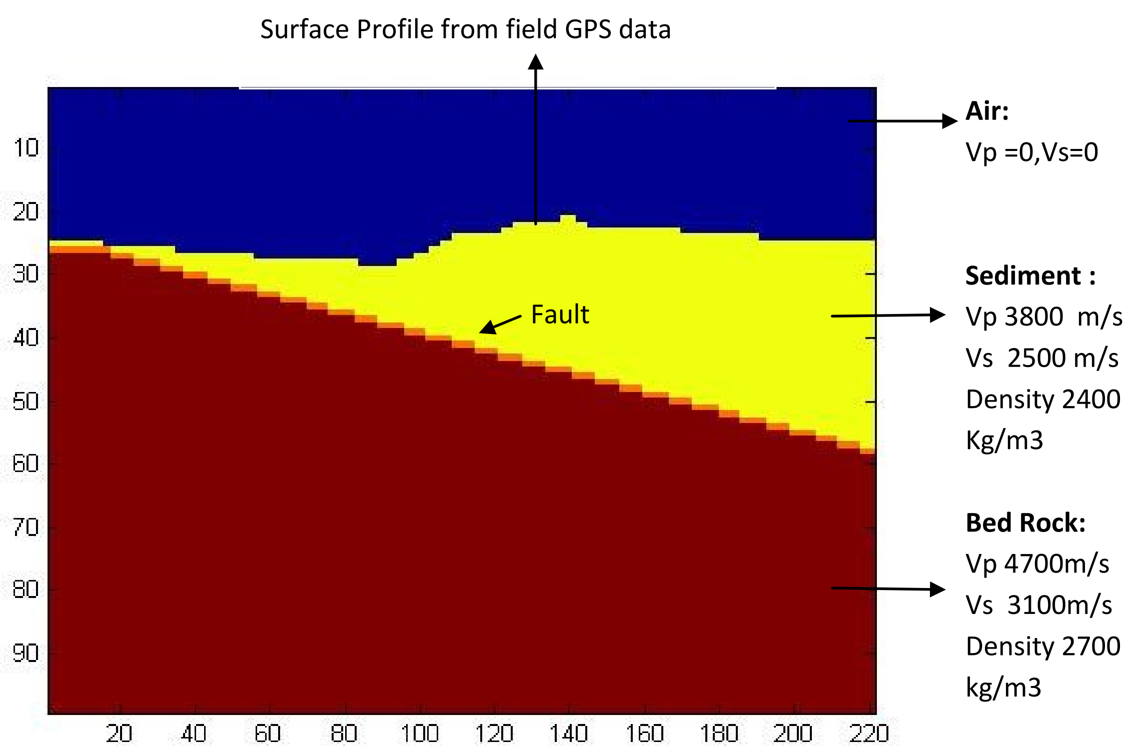
Air Above: Where ever air is present above free surface, it is 20 grid points thick

Material Properties: For the first four cases P wave velocity is 2000m/s, S Wave Velocity is 1200m/s and Density is 2360kg/m3. Model is discussed for the 5th case while the case is presented.

Model for cases 1 to 4:



Model for Case 5:



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1) PML Vs No PML

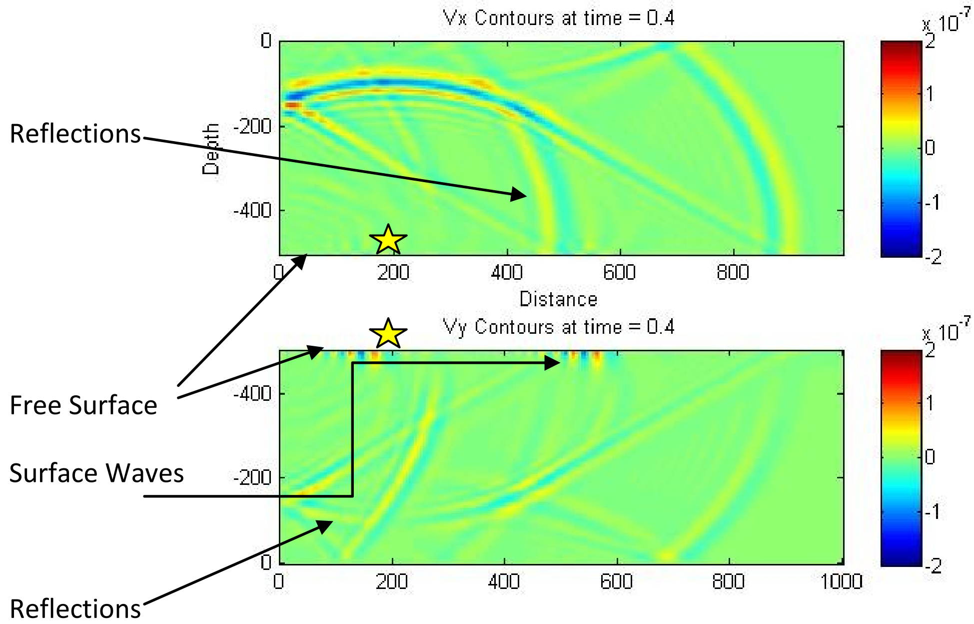

Figure: Simulation without PML BCs. .

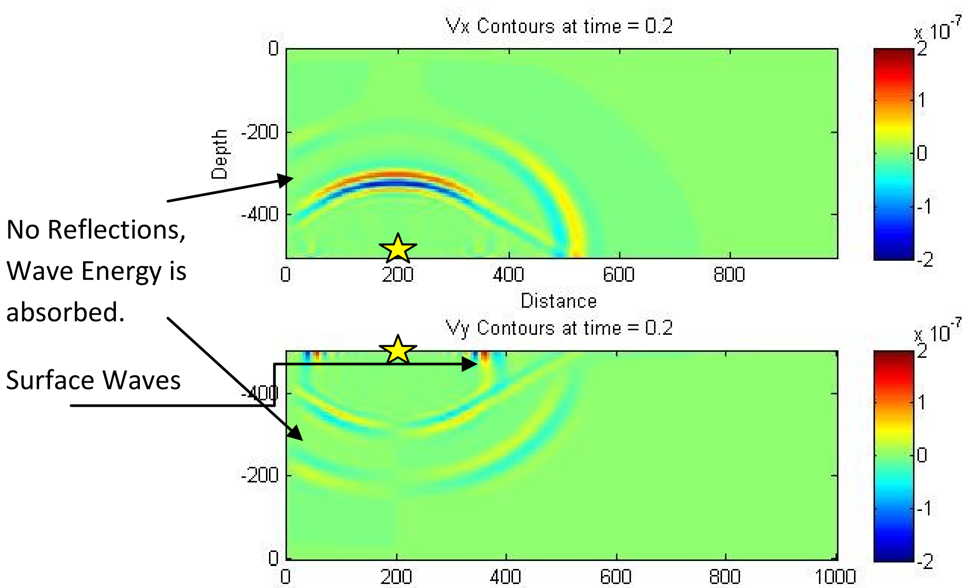

Figure: Simulation with PML BCs. .

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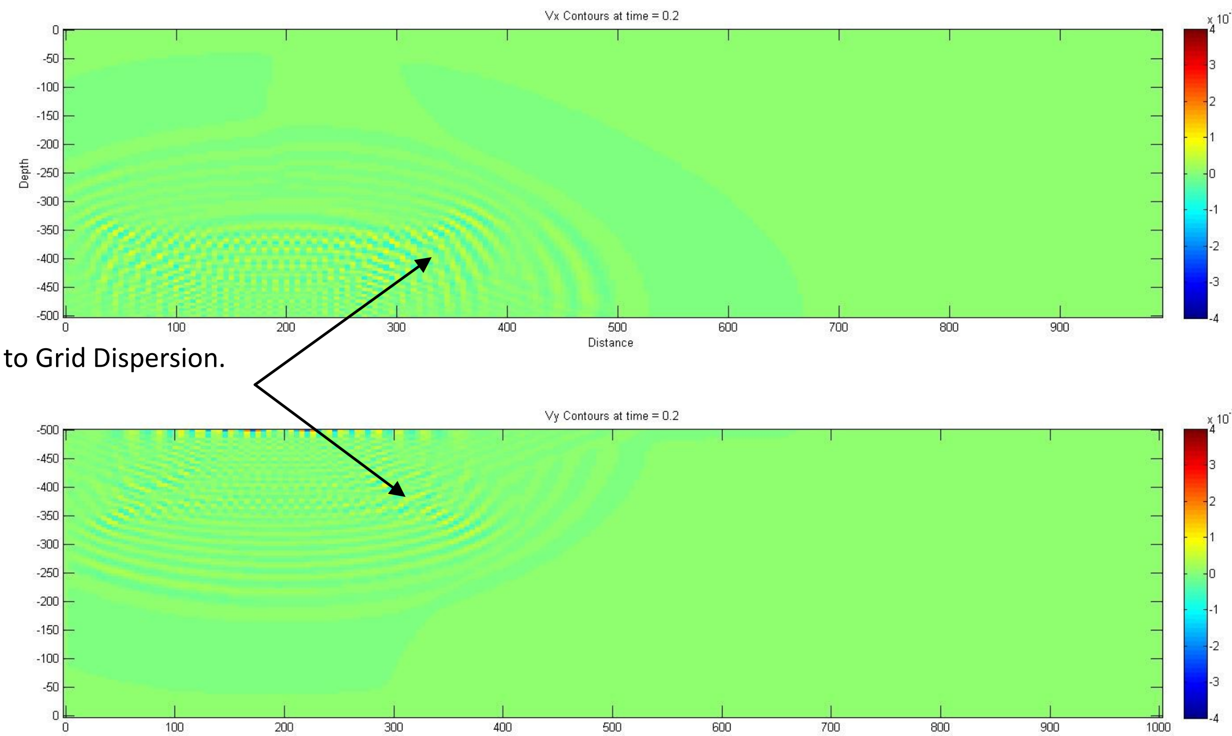
Discussion:

1) We can see the generation of Surface waves and reflection phenomenon very well.
2) See that PML Boundary Conditions are very efficient in absorbing the wave energy.

2) Effect of Source Frequency and grid Dispersion. .



Artifacts due to Grid Dispersion.

Figure: With 100Hz frequency Source. .

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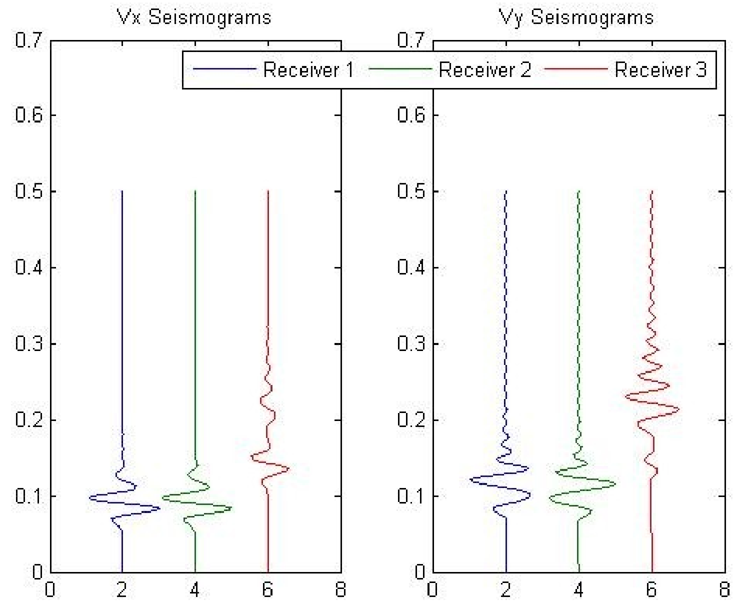
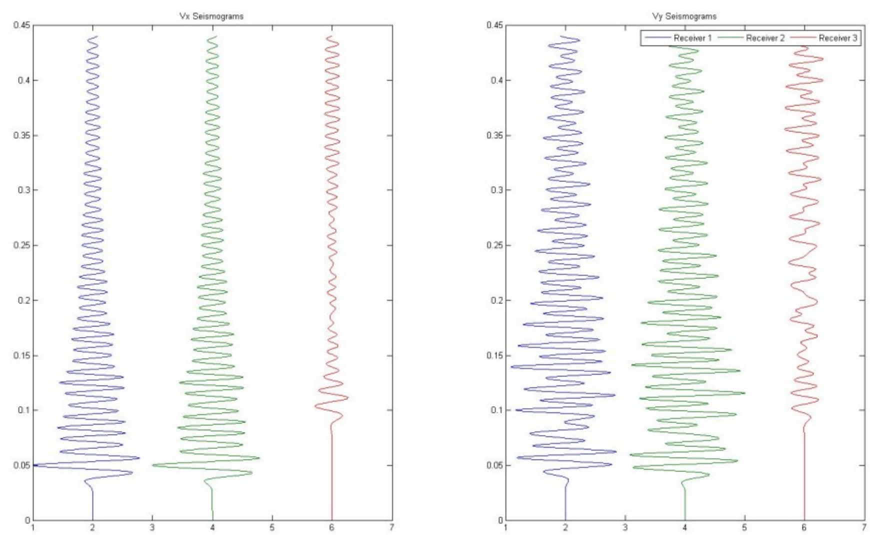


Seismogram for Source Frequency = 25Hz (Free Surface by Image Method)



Seismogram for Source Frequency = 100 Hz

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Discussion:

1) We have used a frequency of 100Hz, whose wavelength is 1200/100 = 12. But, we need 12/10 = 1.2m grid spacing to accurately simulate such high frequency. We have grid spacing of 5m only.
2) Observe the seismograms presented. We almost reproduce the input signal in the low frequency (25Hz). But, at 100 Hz, we have many arrivals. This is due to the arrival of different frequencies at different times.They are all not travelling with the same velocity- Dispersion phenomenon

3) Comparison between vacuum and Image Method

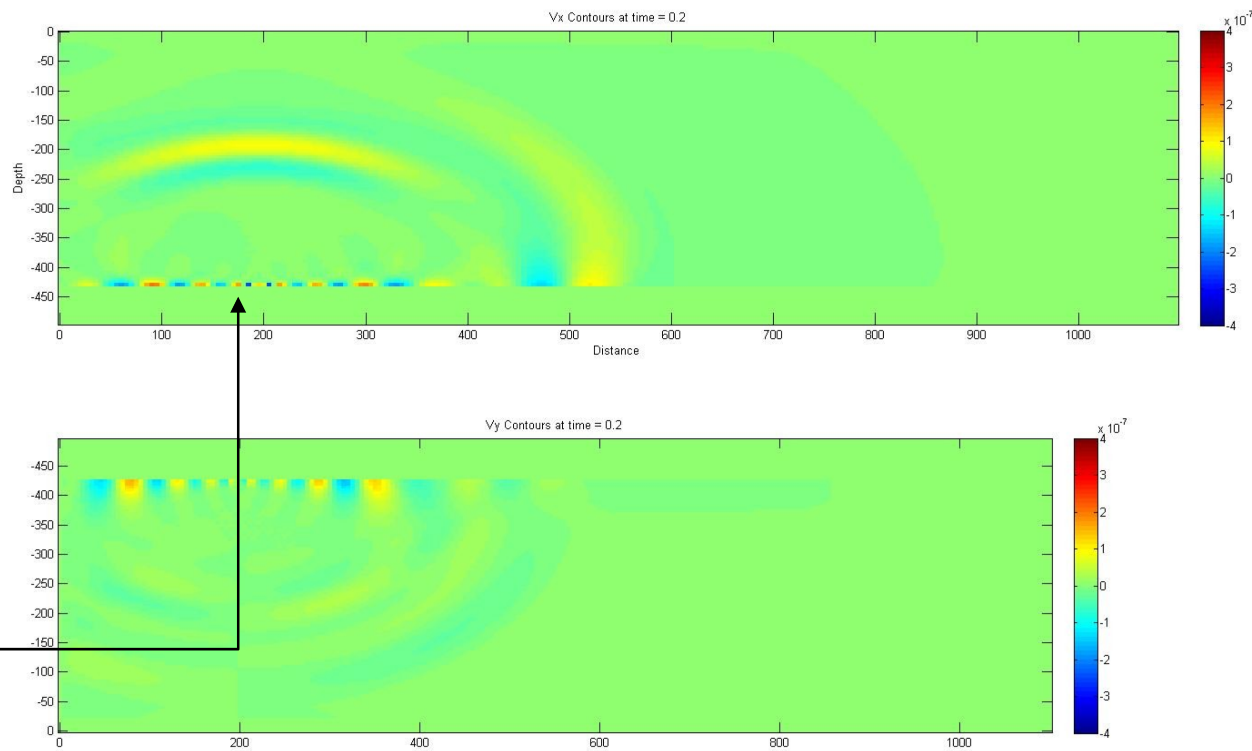

Figure: Simulation with Vaccum Condition for free Surface at 25Hz Source

Observe that energy is trapped at the surface. Though it looks like surface wave it is not the surface wave. Compare it with 25Hz figure image BC earlier. We don't see those spots

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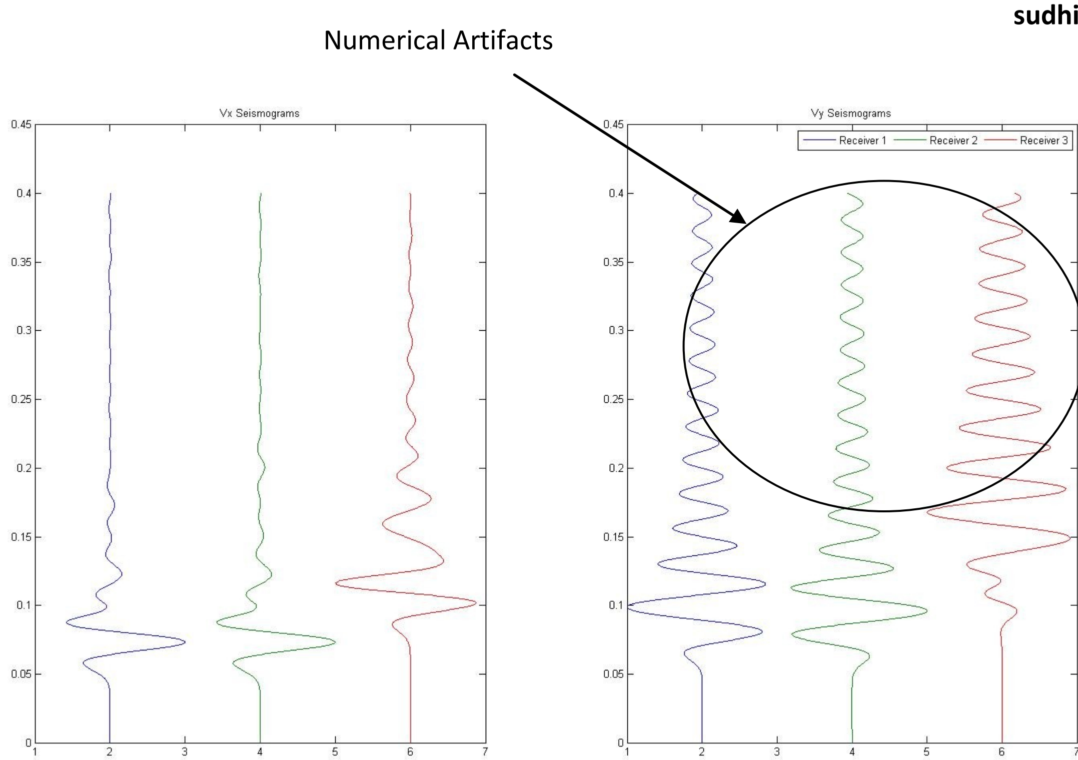

Figure: Seismogram for Vacuum Free Surface Condition at 25Hz Source

Discussion:

1) We can clearly see that Vacuum Method is not so robust. Energy is being trapped at the surface. And also when run for very long times, instability develops at the surface.
2) From the seismogram, comparing it to image method, we see that vacuum Method adds some numerical noise to the seismogram. So, ideally, if we have an alternative, Vacuum Method is not the right choice to go for to model Surface topography. .

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4) Multi-Grid Method: **.**

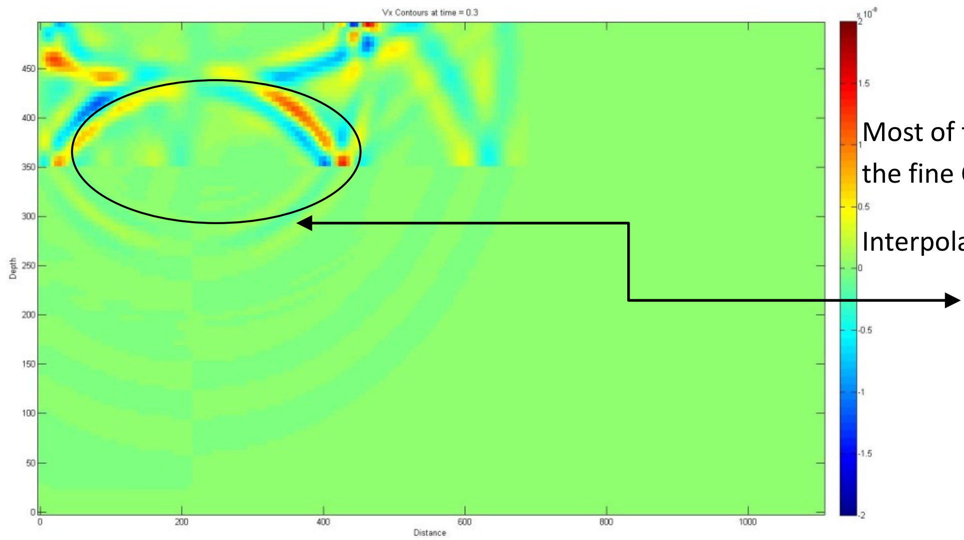


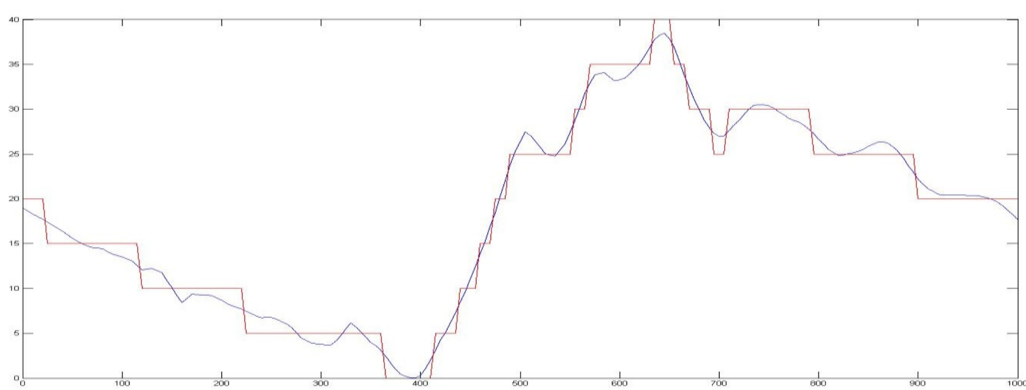
Figure: Simulation with multi Grid.

Discussion:

1) Though Multi-Grid would come to great help in saving memory, the artificial reflections created by the fine coarse –grid interface are always painful. They would disturb the entire simulation if not taken care properly.

5) **Surface Topography:**

The GPS measurements of the Topography present at the field camp is shown below:



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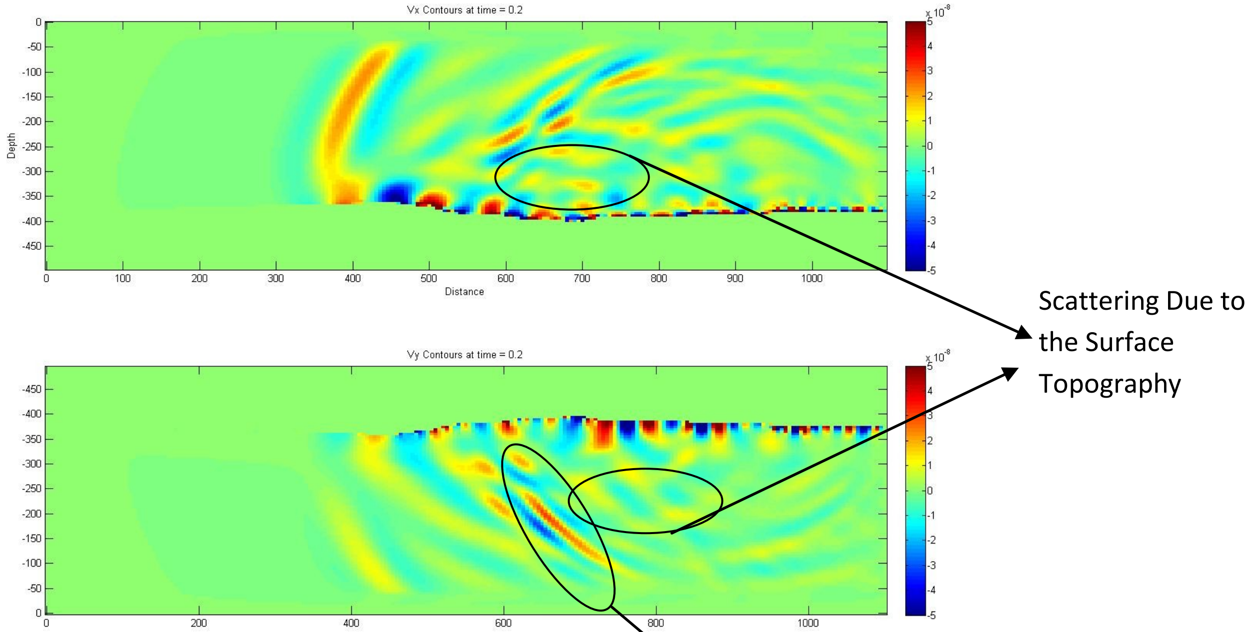

Figure: Simulation after Introducing Topography (Fault Not introduced yet)

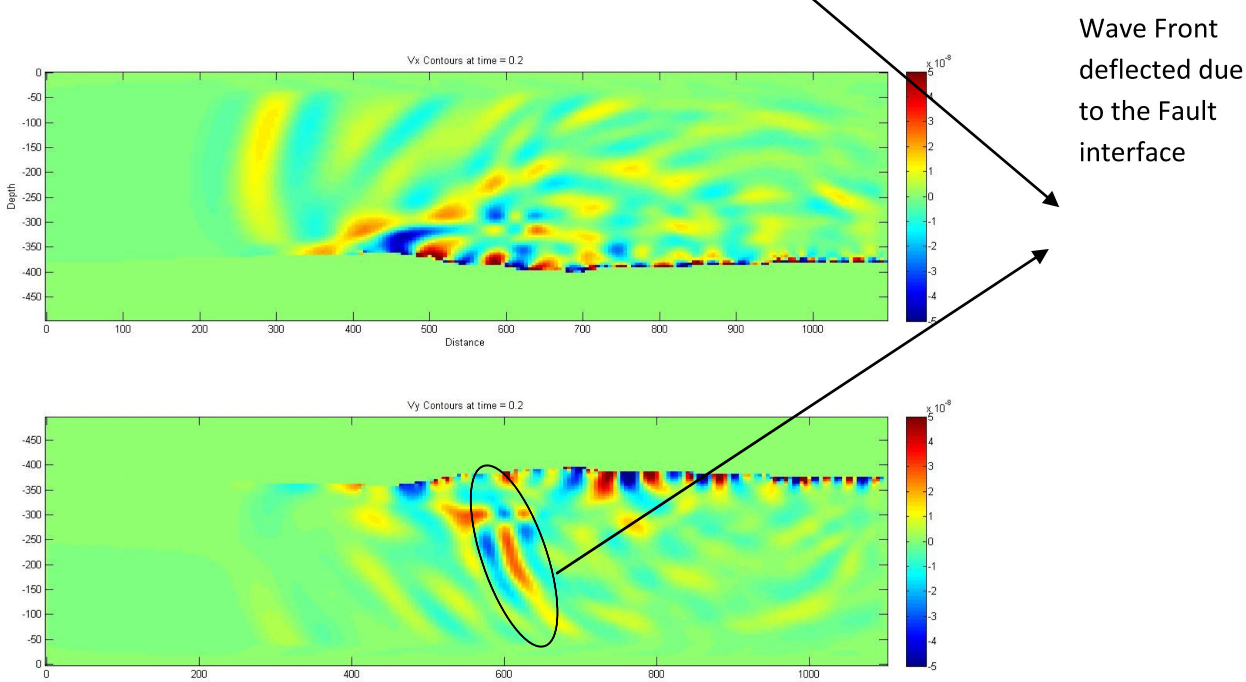

Figure: Simulation after Introducing Topography and Fault

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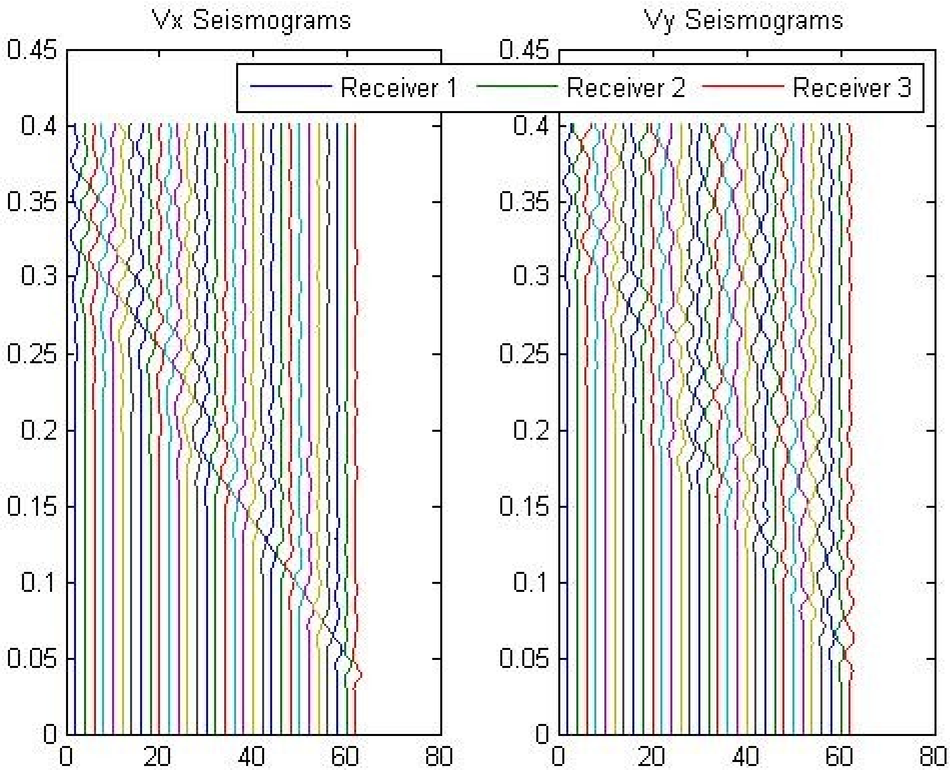

Discussion:

1) We can observe the Surface Scattering Phenomenon. But still we see the energy being concentrated at the surface, which now we cannot judge if it is due to numerical artifact or due to topography effect.

2) We can also observe that the wave front is deflected by the Fault present. This is according to the snells law which we would physically anticipate.

7. Conclusions:

1) Vacuum Method is easy to CODE. But, it introduces numerical artifacts. And is unstable. The Imaging Method is theoretically robust. .
2) Frequency of source matters a lot in what you see. If you use very high frequency more than what your grid can handle, it would be a mess. .
3) Multi-grid method though helps you to reduce the computational expense, produces numerical reflections which you don't want to. Interpolation is very crucial in Multi-Grid. Interpolation is crucial in multi-gris methods. Orelse you pay with large reflections which spoil your simulation. .
4) Understanding source mechanism is crucial. You get entirely different results if source behavior is different.

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8. Future Work

1) I have implemented Surface topography by vacuum Method here. But, the robust way is to develop the imaging algorithm for Imaging Method. Future work would be focused on developing Code to handle Surface topography by Imaging Method.
2) I have implemented Multi-Grid method to plane Free Surface Condition. It should be extended to Topography.
3) Explore effect of Source in the Seismic Signal recorded and search for better alternatives.

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10. **Appendix**

CODE attached in email