18.085 SUMMER 2013 - QUIZ 3 - AUGUST 16, 2013

**YOUR NAME:** ______________________________

**YOUR SCORE:** ____________ **/ 100 +** __________ **/ 20 extra credit**

**THE QUIZ IS OPEN BOOK, OPEN NOTES AND NO CALCULATORS**

**GRADING:**

(1) **1.** ______________

(2) **2.** ______________ + ______________ **BONUS POINTS**

(3) **3.** ______________

(4) **4.** ______________

(1) (20 points.)
The dft of the signal $x$ is

$$\hat{x} = \begin{pmatrix} 3 \\ 0 \\ 2i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2i \\ 0 \end{pmatrix}$$

What signal is sampled in $x$?
Sketch the real and the imaginary part of $x$. (With our usual notation, here we have that N=11).

(2) ($15+15+10$ points $+10+10$ bonus points.)
Which of these equations can be solved? If you can solve them, then show a solution, otherwise explain why it is not possible to find a solution.

a)
$$\operatorname{div}\begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{pmatrix} = y^2 - 3\lambda x - 1,$$
where $\lambda$ is a fixed real parameter.

b) The Laplace equation on the unit circle, with boundary condition,
$$-\Delta u = 0, \; u(1,\theta) = \cos(2\theta) + \sin(6\theta) + 1.$$

c)
$$\operatorname{div}\begin{pmatrix} -\frac{\partial s}{\partial y} \\ \frac{\partial s}{\partial x} \end{pmatrix} = e^{x^2-3y},$$

**Caveat:** the next two are bonus questions.

d1) Find a family of curves $s(x, y) = C$ that is everywhere orthogonal to the family of curves $u(x, y) = e^{-y}(\sin(x))$.

d2) Can you tell for which complex function the function $u(x, y)$ in d1) is the real part?

(3) (15 points). Let $f(x) = -|x| + g(x)$ on $[-\pi, \pi]$, where $g(x)$ is defined as follows,

$$g(x) = \begin{cases} 12 & \text{for } x \in [0, \pi] \\ -2 & \text{for } x \in (-\pi, 0). \end{cases}$$

Find the Fourier series of $f(x)$, when $f$ is extended periodically to the real line, by the rule $f(2\pi + x) = f(x)$.

(4) (5 + 5 + 5 + 5 + 5 points)

a) Consider the function $f(x) = e^{-x},\ x \in [-\pi, \pi)$ which is extended periodically to the real line, by the rule $f(2\pi + x) = f(x)$. Draw the graph in $[-2\pi, 2\pi]$.
The functions satisfies the following differential equation

$$\frac{d}{dx} f(x) + f(x) = g(x), \tag{0.1}$$

for some function $g(x) : [-\pi, \pi] \to \mathbb{R}$. Find such $g$.

b) Compute the complex Fourier series of $f(x)$,

$$f(x) = \sum_{k \in \mathbb{Z}} d_k e^{ikx},$$

in the standard way.

c) Try to compute the Fourier coefficients of $f$ using the differential equation (0.1). Do you get the same result as in part b?
d) Explain the reason for the answer you gave in part c.
e) Compute

$$\sum_{k\in\mathbb{Z}} |d_k|^2.$$