18.085 SUMMER 2013 - QUIZ 2 - JULY 26, 2013

YOUR NAME: ______________________________

YOUR SCORE: ____________ **/ 100 +** ____________ **/ 5 extra credit**

(1) ($5 \times 6 = 30$ points.)

Consider the truss drawn on the board.

a) Is there a possible mechanism or rigid motion? If so, draw one.

b) Build the matrix A corresponding to this truss.

c) Use the matrix A to verify mathematically your answer to (a). That is, depending on your answer to (a) : either prove the nullspace of A is trivial, or give a basis for the nullspace of A (no need to prove you have indeed a basis, but be sure of your answer!).

(Problem 1 continued.)

d) Now fix node 2. Get rid of bar 3 since it is now useless. What is the matrix A for this new truss?

e) Is the truss in (d) stable? Prove this mathematically.

(2) ($3 \times 8 + 2 \times 3 = 30$ points.)

Consider a hanging bar. We have the following equation for its displacement u, given that $c(x) = 1/2$ for $x < 1/2$ and $c(x) = 2$ for $x > 1/2$, and $f(x) = \delta(x - 1/2)$:

$$-\frac{d}{dx}\left(c(x)\frac{du}{dx}\right) = f(x), \qquad x \in [0,1].$$

We also have the boundary conditions $u(0) = 0$ and $\frac{du}{dx}(1) = 0$.

a) Find $w(x) = c(x)\frac{du}{dx}$, and graph it.

b) Find $\frac{du}{dx}$, and graph it.

c) Find u, and graph it.

Circle your answers to (d), (e) (no explanation).

d) Which part of the bar is stiffer: $0 < x < 1/2$ or $1/2 < x < 1$.

e) Which part of the bar is stretched (more): $0 < x < 1/2$ or $1/2 < x < 1$.

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(3) ($6 + 6 + 5$ extra $+12 = 24$ points $+5$ extra credit points.)

A system of springs and masses can be modeled using $M\vec{u}'' = -K\vec{u}$ for $\vec{u}$ the vector of displacements of the masses. We use primes to mean time derivatives. $K = A^T CA$, where$A$ is the first difference matrix and C is the constitutive law.

a) What does the equation $M\vec{u}'' = -K\vec{u}$ for $\vec{u}$ simplify to if we only have one mass and one spring? (Use the notation on the board.)

b) Find an analytical solution to the equation $mu'' = -ku$, for $u = u(t)$. Don't bother with initial conditions. Just give ONE expression (there could be more than one that work, it could be complex) for u which satisfies the given equation.

c) EXTRA CREDIT QUESTION, OPTIONAL. Now find a solution of $mu'' = -ku$, for $u = u(t)$, with the following initial conditions: $u(0) = 1$ and $u'(0) = 0$.

d) We want to solve the problem in (c) with $m = 1$ (so we will ignore m) and $k = 4$ using the leap-frog method (finite differences in time). Modify lines 4, 5 and 9 (don't modify anything else) of the following Matlab code to do this.

```
01: T=2*pi;nt=50;dt=T/nt;
02: k=4;
03: u=zeros(1,nt+1);
04: u0=
05: up0=
06: u(1)=u0;
07: u(2)=u0+dt*up0;
08: for i=3:nt+1
09:     u(i)=
10: end
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(4) ($6 + 10 = 16$ points.)

(a) I want to use Newton's method to solve $p(x) = 0$ with starting guess x_0. Write down the algorithm to find the next guesses x_1, x_2, etc (1 line, plus say for which indices that line holds).

(b) I want to solve Laplace's equation $-\frac{d^2u}{dx^2} = f$ on $x \in [0, 1]$ with boundary conditions (CAREFUL!) $u(1) = 0$ and $\frac{du}{dx}(0) = 0$. Let $h = 1/3$. Draw the hat functions you would use to solve this problem using the Finite Element Method. Label your axes. OK to draw them all on the same graph.