- 1.6.20

The matrix A is of the form $Q\Lambda Q^T$, where Q is orthogonal.
a) $\det A = 2 * 5 = 10$.
b) The eigenvalues of A are 2 and 5.
c) $(cos(\theta), sin(\theta))^t$ and $(-sin(\theta), cos(\theta))^t$ are the two eigenvectors.
d) since $A = Q\text{diag}\{2,5\}Q^t$, which is a symmetric matrix and has two positive eigenvalues, it is positive definite.

- 1.6.22

Let H= $\begin{pmatrix} \frac{\partial^2(z)}{\partial(x)\partial(x)} & \frac{\partial^2(z)}{\partial(x)\partial(y)} \\ \frac{\partial^2(z)}{\partial(y)\partial(x)} & \frac{\partial^2(z)}{\partial(y)\partial(y)} \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2b & 2c \end{pmatrix}$, then $(0,0)$ is saddle if and only if it has one positive and one negative eigenvalue which is equivalent to $\det(H) < 0$, i.e. $ac < b^2$.

- 1.6.24

Just simplify the right hand side to get the left hand side. Remember that K is symmetric.
The missing word is "non-negative".

- 1.6.27

a) Let $H = S\Gamma_H S^t, K = T\Gamma_K T^t$, then $M = diag(S,T)diag(\Gamma_H, \Gamma_K)diag(S^t, T^t)$ which is symmetric and positive. However, N is not, because $det(N) = 0$ (two lines equal).

b) The eigenvalues of M are the union of eigenvalues of H and K. The eigenvalues of N are the eigenvalues of K multiplied by 2 and the rest of the eigenvalues are zeros .
The pivots of M are the union of pivots of H and K. The pivots of N are the pivots of K and the rest is zeros.

c) $chol(M) = diag(chol(H), chol(K))$;

• 2.1.1

a) $c_1c_2c_3$;
b) 0

• 2.1.3

Suppose M=A^tCA=$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$.

we have $(A^t)^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$,

so $M^{-1} = A^{-1}C^{-1}(A^{-1})^t$= $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

• 2.1.7

When $c_1 = c_3 = c_4 = 1, c_2 = 0$, we have $K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ which has inverse $K^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. so $u = K^{-1}f$, which is $(1, 3, 2)^t$.

• 2.1.8

a) $K_{free-free} = \begin{pmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}$

$= \begin{pmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}$.

b) $K_{free-fixed} = \begin{pmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}$

$= \begin{pmatrix} c_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix}$

c) $K_{fixed-fixed} = \begin{pmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3+c_4 \end{pmatrix}$

$$= \begin{pmatrix} c_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c_4 \end{pmatrix}.$$