Quiz 2

18.085 (Prof. Edelman)

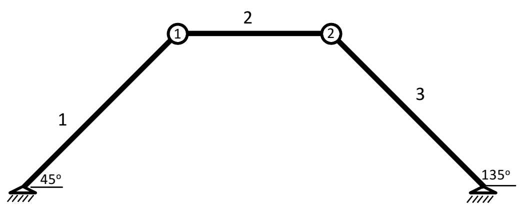
April 9, 2010

PRINTED NAME:____

CLASS NUMBER (the number you use when submitting hw):____________________

- Do all your work on these pages. No calculators or computers may be used. Notes and the text may be used. The point value (total is 33) of each subproblem is indicated.

1. (14 points total)



This problem concerns the truss above with bars labeled 1,2, and 3. Nodes 1 and 2 in the circles have displacement vectors (u_1^H, u_1^V) and (u_2^H, u_2^V), respectively. The other two nodes are fixed.

a. (5 points) Find the elongation matrix A so that

$$e = A \begin{pmatrix} u_1^H \\ u_1^V \\ u_2^H \\ u_2^V \end{pmatrix},$$

describes the stretch in bars 1, 2, and 3.

b. (5 points) What are the nullspace solutions to Au=0? Sketch the motion(s) corresponding to these solutions.

c. (1 points) How many bars do you need to add to eliminate these motions?

d. (3 points) Where would you add any new bars, and what is the new matrix A?

2. (10 points total) This problems concerns the Boundary Value Problem

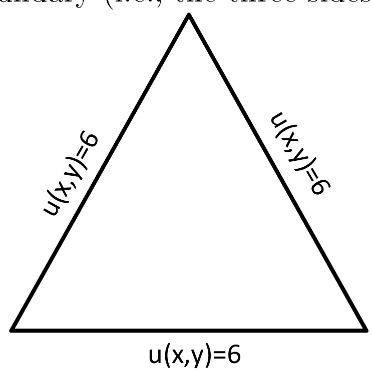
$$\begin{cases} -u''(x) = \delta(x - 1/2), \\ u(0) = u(1) = 0. \end{cases}$$

a. (5 points) If we let $h = 1/3$, and use the piecewise linear hat functions ϕ_1 and ϕ_2 as in your text, we can set up a finite element equation $KU = F$. Calculate the vector F. (Note as usual the hat functions are used both for U and V.)

b. (5 points) The finite element solution to the above problem gives the same answer as a finite element solution to a Boundary Value Problem that does not involve any delta functions. Write down that Boundary Value Problem fully. Explain briefly but convincingly why the solution is the same.

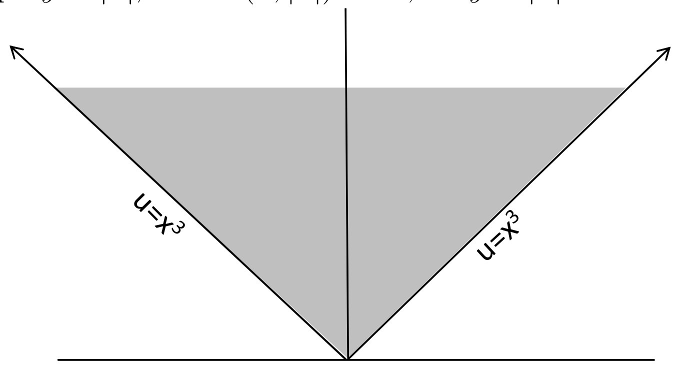
3. (9 points total) These are three separate problems related to solutions to Laplace's equation.

a. (2 points) Find a solution u to Laplace's equation inside an equilateral triangle whose boundary (i.e., the three sides) has the value $u(x, y) = 6$.



b. (3 points) Find a solution u, to Laplace's equation inside the unit circle whose boundary has the value $u_0(\theta) = \sin(2\theta + \pi/3)$.

c. (4 points) Find a solution u, to Laplace's equation on the infinite "wedge" above the graph $y = |x|$, with $u(x, |x|) = x^3$, on $y = |x|$.



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