Your PRINTED name is:	SOLUTIONS	1
Your class number is	O	2.
		3.
		4.

(1) (27 points) This question is about the  $2\pi$ -periodic one-sided pulse f(x):

$$f(x) = e^{-ax} \text{ for } 0 \le x \le \pi, \ f(x) = 0 \text{ for } -\pi < x < 0.$$

(a) Find the Fourier coefficients 
$$c_k$$
 in  $f(x) = \sum_{-\infty}^{\infty} c_k e^{ikx}$ .

$$C_K = \frac{1}{2\pi i} \begin{cases} e^{-(\alpha + iK)x} \\ e^{-(\alpha + iK)x} \end{cases} = \frac{1}{2\pi i} \begin{cases} e^{-(\alpha + iK)x} \\ e^{-(\alpha + iK)x} \end{cases}$$

(b) What is the decay rate of  $c_k$  as  $k \to \infty$ ? What are all the singularities of f(x) that produce this decay rate? You could graph f(x).

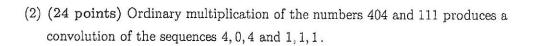
The docay rate is it

F(K) has TWO imps (jump of K=0)

-3 for missing 2nd jump (jump at K=± 17)

(c) As  $a \to 0$ , what limit function F(x) do you get? What are the Fourier coefficients of F(x)?

As a >0, F(x) expressed F(x) = Squar wave Fourier coefficients = {0-mexed



(a) What sequence comes out of that convolution (not cyclic)? Where does 1+1+1=3 multiplied by 4+0+4=8 appear in the

(b) The solution to the differential equation dy/dx = ay + s(x) with y(0) = 0 is  $y(x) = \int_0^x e^{a(x-t)} s(t) dt = \underbrace{e^{ax} * s(x)}_{in} \text{ in my method}$ 

Compute dy/dx to verify that this y(x) solves the differential equation.  $y(x) = e^{ax}(\int x - at s(t) dt) = product of two$   $dy = ax \left( - at s(t) dt + e^{ax} \left( - ax s(x) \right) \right)$   $dy = ae^{ax} \left( - at s(t) dt + e^{ax} \left( - ax s(x) \right) \right)$   $dx = ae^{ax} \left( - at s(x) \right)$ This is  $s(x) = e^{bx}$ , compute y(x).

Theorem of

Colculus:

Derivative & Integral

Qives original function

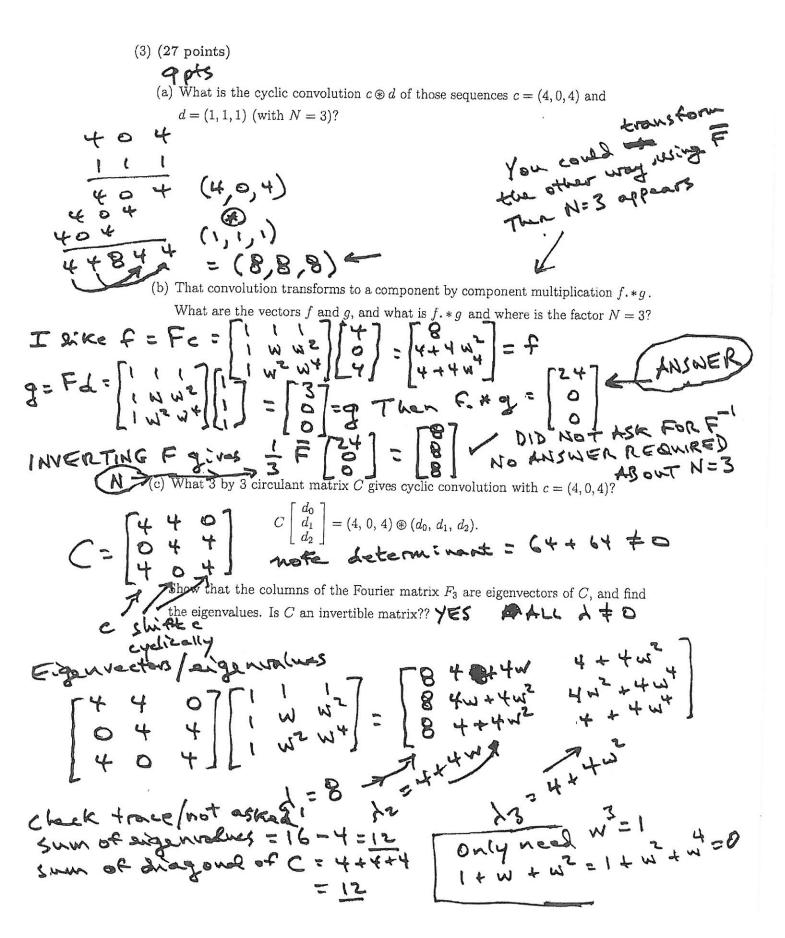
ax (b-a) + | Must Mse this somewhere

since we don't know s(x)

ax (b-a) x ax

bx ax

6 points



(4) (22 points) Suppose I want to use the Fourier integral transform to solve this fourth-order differential equation:

$$\frac{d^4u}{dx^4} = \delta(x) - 4\delta(x-1) + 6\delta(x-2) - 4\delta(x-3) + \delta(x-4).$$

(a) Take the transform of all terms to find  $\widehat{u}(k)$ .

(+ik)  $\widehat{u}(k) = 1 - 4e^{-ik} + 6e^{-2ik} - 4e^{-3ik} + e^{-4ik}$ (i'+i)  $= (1 - e^{-ik})^{4} + 2e^{-4ik}$ 

Divide by  $K^+$  to get  $W^+$ )

(b) The solution u(x) will have jumps in which derivative (?) at which points x(?).

Tumps of 1, -4, 6, -4, 1 in the third solvative atx=0,1,2,3,4

(c) What is u(x) between x = 0 and x = 1? Do you recognize u(x)?

W=0 for megative x. I make ph 1. Then w= x from x=0 to x=1

it is a cubic B-spline (with continuous) It is sprofer XED and X34