18.085 Computational Science and Engineering	Sep 29,2009

Solutions to Problem Set 2

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Note: Solutions for 1.4-7, 1.5-1,7,9 and 1.6-6 can be found in the Opencourseware. Besides, solutions for 1.3-9,11 (Matlab practice) will not be provided here.

[1.3-1]

$$K_4 = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 \\ 0 & 0 & 4/3 & 0 \\ 0 & 0 & 0 & 5/4 \end{pmatrix} \begin{pmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 1 & -3/4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

And we have $\text{Det}(K_4)$ = product of its pivots = 5.

[1.3-5]
The pivots of B are 2 and 7. Since A has pivots 2,7,6 with no row exchanges, the first two pivots will not be affected when applying Gaussian elimination.

[1.3-9]
For $chol(B_3 + eps * eye(3))$, the last term in the main diagonal should not be zero, but a very small number instead.

[1.4-1]
Let $u(x) = Ax + B$ for $0 \le x \le a$ and $u(x) = Cx + D$ for $a \le x \le 1$. The first two conditions are the given initial conditions $u(0) = 2$ and $u(1) = 0$. The third is $Aa + B = Ca + D$ by the continuity at $x = a$. Finally, we have $A - 1 = C$ by the change in slope. Solving these conditions, we have

$$\begin{aligned} A &= -(a+1) \\ B &= 2 \\ C &= -(a+2) \\ D &= a+2 \end{aligned}$$

[1.4-4]
Let $u(x) = Ax + B$ for $0 \le x \le a$ and $u(x) = Cx + D$ for $a \le x \le 1$. By the given initial conditions, we have $u(0) = 0$ and $u'(1) = 0$. Then by the continuity at $x = a$ and the

change in slope, we have $Aa + B = Ca + D$ and $A - 1 = C$. Solving these conditions, we have $u(x) = x$ for $0 \le x \le a$ and $u(x) = a$ for $a \le x \le 1$. (Graphs will not be provided here.)

[1.4-15]

$$\begin{aligned}\int_{-\infty}^{+\infty} g(x)\delta'(x)dx &= g(x)\delta(x)|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \delta(x)dg(x) \\ &= 0 - \int_{-\infty}^{+\infty} \delta(x)g'(x)dx \\ &= -g'(0)\end{aligned}$$

[1.5-13]

$$A + 2I = S\Lambda S^{-1} + S(2I)S^{-1} = S(\Lambda + 2I)S^{-1}$$

So the eigenvector matrix for $A + 2I$ is still S.

[1.6-4]
By row reductions, we have

$$\begin{aligned}C &= \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & -1 \\ 0 & 3/2 & -3/2 \\ 0 & 0 & 0 \end{pmatrix} \\ &= L^T U\end{aligned}$$

So the pivots of C are $2, \frac{3}{2}$ and 0, all are positive or zero. Therefore the matrix is semidefinite. Then we have

$$u^T C u = 2(u_1 - \frac{1}{2}u_2 - \frac{1}{2}u_3)^2 + \frac{3}{2}(u_2 - u_3)^2 + 0u_3^2$$

where the coefficients $2, \frac{3}{2}, 0$ are the pivots of C, and the coefficients in the linear forms $u_1 - \frac{1}{2}u_2 - \frac{1}{2}u_3$, $u_2 - u_3$ and u_3 are the columns of the reducing matrix L^T above.