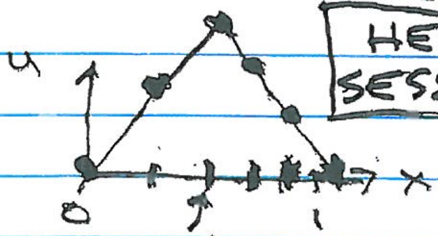
Notes on 18.085 class / Tuesday afternoon

SECTIONS 1.2, 1.4 Sept 10

① Solution of $Ku = \delta_j$ = point load = $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ at node $j=2$

HELP WED SESSION 4-5 IN 4-149 TODAY 9/11



$x = a = 2h = \frac{2}{5}$

discrete u falls exactly on

continuous $u(x) = \begin{cases} (1-a)x & x \le a \\ a(1-x) & x \ge a \end{cases}$

Discrete solutions for loads at meshpoints 1, 2, 3, 4 are the columns of the matrix K^{-1} !! (Figure 1.9)

$$\begin{bmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix} \begin{array}{c} 0 \quad 0 \quad 0 \quad 0 \\ \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 6 & 4 & 2 \\ 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix} \\ 0 \quad 0 \quad 0 \quad 0 \end{array} \times \frac{1}{5} \qquad u = \frac{1}{5}\begin{bmatrix} 3 \\ 6 \\ 4 \\ 2 \end{bmatrix}$$

column 2 from load 2 : linearly up $0 \ \frac{3}{5} \ \frac{6}{5}$ / $\frac{6}{5} \ \frac{4}{5} \ \frac{2}{5}$ down to 0.

We could check this solution

Discrete solution from cts solution for $-u'' = \delta(x-a)$

Notice that K^{-1} is always symmetric if K is

Transpose $KK^{-1} = I$ $\quad (K^{-1})^T K^T = I$, $K^T = K$ $\quad$ THEN $K^{-1} = (K^{-1})^T$

② Free-fixed boundary conditions

free end has $\frac{du}{dx} = 0$ instead of $u = 0$

Discretely we could choose between

$u_1 = u_0$ OR $u_1 = u_{-1}$

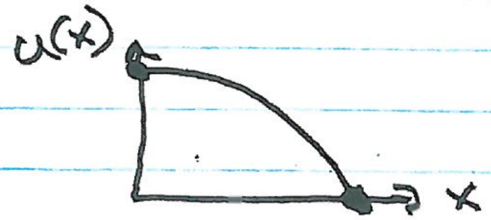
↗ TODAY ↖ FUTURE

Continuous problem / free-fixed / uniform load

$$-u''=1 \qquad \begin{matrix} u'(0)=0 \\ u(1)=0 \end{matrix} \qquad u = A + Bx - \frac{x^2}{2}$$

$$B = 0$$

$$A = \frac{1}{2} \qquad \underline{u = \frac{1}{2}(1-x^2)}$$



$$-u'' = \delta(x-a) \qquad u'(0)=0 \longrightarrow B=0$$

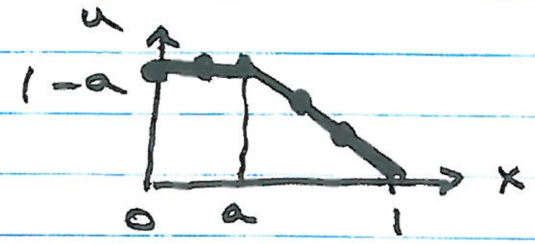
2-part solution $\qquad u(1)=0 \longrightarrow C+D=0$

$$u = \begin{cases} A + \cancel{Bx} & x \le a \\ C + Dx & x \ge a \\ (1-x) & \end{cases} \quad \text{agree at } x=a \quad A = C(1-a)$$

SLOPE DROPS BY 1

From slope 0 to slope $-1 = D$

$$\text{Altogether } u = \begin{cases} 1-a & a \le x \\ 1-x & x \ge a \end{cases}$$



no stretching above load

uniform compression below load

K matrix changes to T matrix

$$\overset{T}{\begin{bmatrix} 1 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix}} \overset{T^{-1}}{\begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

impulse responses $\qquad$ ↑ impulses

Key point to come, for uniform load

Discrete solutions to $Tu = \text{ones}$ DO <u>NOT</u> LIE on continuous solution $u = \frac{1}{2}(1-x^2)$ to $-u''=1$, $u'(0)=0$ $u(1)=0$