

# NOTES FROM TUESDAY SEPT 17 3-4

①  $A^T A$  is at least positive semidefinite  
If  $A$  has independent columns,  $A^T A$  is

positive definite

this means that:

$Ax=0$  has only the solution  $x=0$  (zero vector)

Reasoning Use the energy test for pos <sup>semidef</sup> def

$$x^T A^T A x \geq 0 \text{ WHY?}$$

$(Ax)^T (Ax) =$  length squared of the vector  $Ax$   
Automatically  $\geq 0$ .

Zero energy when  $Ax=0$ . If  $A$  has indep. columns then  $x=0$ . Energy test passed.

② Four big pbms of linear algebra

A. Solve  $Ku=f$  by elimination

B. From indep vectors  $v_1, \dots, v_r$  compute orthogonal vectors  $q_1, \dots, q_r$

C.  $Kx = \lambda x$ . The eigenvectors diagonalize  $K$

D.  $Av_i = \sigma_i u_i$  —  $Av_i = \sigma_i u_i$  (The SVD)

"singular vectors"  $v$ 's and  $u$ 's diagonalize any  $A$ .

We can choose  $v$ 's = e vectors of  $A^T A$  : orthogonal  
 $u$ 's = e vectors of  $AA^T$  : orthogonal

$$A = U \Sigma V^T$$

If  $A$  was symmetric pos def page 2  
 then  $v$ 's =  $u$ 's = eigenvectors  $\lambda$ 's

If  $A$  is rectangular we must go to  $A^T A$   
 $A A^T$   
 same eigenvalues  $\lambda^2 \geq 0$

## Section 1.8 : Applications of the SVD

THIS IS NOT AN OFFICIAL PART OF 18.085!  
 OPTIONAL!

### Applications to data matrices (rectangular)

$A =$  Samples 1 to n  
 — each row of  $A$  measures one  
 — property for all the samples  
 — "Principal Component  
 Analysis" = PCA  
 columns =  $\downarrow$  students each row gives  
undergraduate grades

Physics grades on row 1, all students in 1 course  
~~row 1~~ 1 student

[ 82 (71) 93 77 42 100 ] in all courses

row 1 column 2  
 (course 1) (student 2)

71 Physics  
 74 Chem  
 83 Math  
 99 ME  
 80 English

The SVD looks for

- ① a column  $v$
- ② a row  $u$

SMART CHOICE  
 $A^T v = \lambda_{\max} v$   
 $A A^T u = \lambda_{\max} u$

So that the  
 simple matrix

$v$  times  $u^T$  = most important part of  $A$  = most information compressed into  
 $m \times 1$  1  $\times$  n just 2 vectors  $v$  and  $u$