Homework 6.

- 2nd order accuracy
- Networks & the ATCA framework

Question 1

In HmWK 3, Q 2 (c) (v), we found first order accuracy of our finite difference approximation to the solution of

$$-u'' = 1$$

$$u'(0) = 0 \quad , \quad u(1) = 0 .$$

The CSE textbook explains how a very simple change can improve this to second order accuracy (section 1.2). With $h = \Delta x = \frac{1}{5}$ construct this second order finite difference scheme

$$a\, A\, u = F$$

i) What is the number a?

ii) " " " vector u?

iii) " " " " F?

iv) " " " matrix A?

(Hint: use MATLAB code on website to check your answer is 2nd order accurate.)

v) On the same graph, plot the exact solution and the finite difference approximation. Hand in the plot.

vi) If the right hand side is changed from $f(x) = 1$ to $f(x) = x$, what is the new vector F?

vii) With $f(x) = x$, does the finite difference approximation agree exactly with the true solution?

Question 2

Complete this table.

Mechanics	Statistics	Networks
u = displacements $e = Au$ (elongations) $w = Ce$ (Hooke's Law) $f = A^T w$ (force balance)		
$A^T C A u = f$		

Question 3

Use the MATLAB code on the website to make the incidence matrix A, and the system matrix K, for the network in quiz 1.

a) print and hand in the matrix A and K.

b) check $Au = 0$ & $Ku = 0$ for $u = \text{ones}(8,1)$.

c) check row 6 of K.

d) check that $A^T C A$ agrees with $D - W$ (weighted degree matrix & weighted adjacency matrix)

e) Ask MATLAB to find independent sol^{n}s to Kirchhoff's Law with null (A').

Show that the solutions you found on the quiz are linear combinations of the solutions that MATLAB found.

(Hand in answers to (a) & (e).)