Question 1 (positive definite, section 1.6)

a) For what values of b is the matrix

$$\begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$$

positive definite?

b) Use the MATLAB code provided on the course website (or your own) to plot

$$u^T \begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix} u$$

for three values of b where the matrix is

i) positive definite

ii) " semi definite

iii) indefinite.

Hand in 3 plots, with values of b.

Question 2

(sections 1.2 & 1.4
Finite differences, inverses & delta functions)

a) solve

$$\begin{cases} -\dfrac{d^2u}{dx^2} = f(x) = \delta\left(x-\frac{1}{2}\right) \\ u'(0) = 0 \quad \text{and} \quad u(1) = 0 \end{cases}$$

b) With $N=5$ and $h=\frac{1}{N+1}$

write the linear equation that you get by applying finite differences to the problem in (a).
(The equation is of the form $Ax=b$, and you must give A, x and b explicitly.)

Hint: check with the MATLAB code provided on the course website.

c) Put $f(x) = 1$ in (a).

i) solve for $u(x)$ exactly

ii) write the finite difference equation, with a 5×5 matrix $(N=5)$.

iii) on the same graph, plot the exact solution and the finite difference approximation. Hand in the plot.

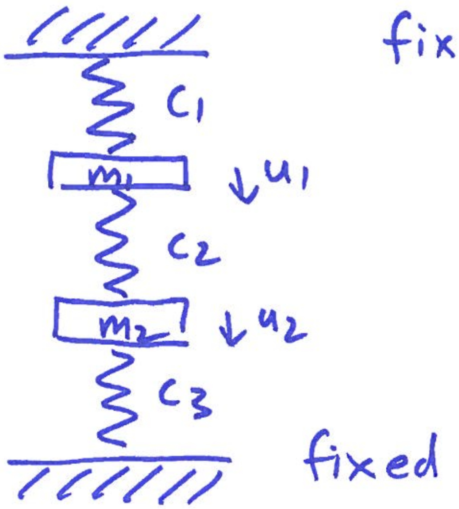
iv) does the finite difference approximation agree exactly with the true solution?

v) What is the order of accuracy as N becomes large?

(Hint: use the MATLAB code provided on the course website, and try LogLog plots).

Question 3

(Sections 2.1 & 2.2 : springs & masses.)



a) Write the matrix A that gives the elongations e of the springs, when applied to the displacements u.

b) Write the stiffness matrix $K = A^T C A$.

c) With $m_1 = m_2 = m = 2$ and $c_1 = c_2 = c_3 = C = 8$ (so downward force on mass due to gravity is mg) and $g = 10$ solve for the displacements $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$.

d) With $M = \begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}$, (and m_1, m_2 and all parameters the same values as before) the equation

$$\boxed{Mu'' + Ku = 0}$$

has natural frequencies of oscillation. What is the fastest frequency? (Hint: use MATLAB eig(K, M) to 1 dec. place.)