Problem Set 3 (Due Feb. 25th)

Questions on least squares will be added later.

1. For a fixed-free line of 3 springs and masses (spring constants c_1, c_2, c_3) write down A and C and $K = A^TCA$.

2. In this "statically determinate" problem with square matrices $m = n$ you can do this: Invert K by inverting its 3 factors A^T, C, A.

   Show that this K^{-1} has all positive entries / all displacements are downward.

3. For one free-free spring sitting between 2 masses, show that the stiffness matrix is

   $$\text{"element matrix } K\text{"} = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}$$

   when $m = 1$.

   Now assemble the three element matrices for Problem 1 into the 3 by 3 matrix K for Problem 1.

   The first element matrix will only be 1 by 1 because spring 1 is fixed at the top.

4. Suppose spring 2 gets weak and c_2 goes to zero. What is the limiting K when $c_2 \approx 0$? What are the properties of this limiting K? With $c_2 = 0$ and $c_1 = 1$ and $c_3 = 3$, find the eigenvalues of K. (OK to use `eig(K)` if you want.)

5. Let's introduce random spring constants and run a number of simulations. Choose c_1 and c_2 and c_3 each as `2 + rand(1)` in 100 simulations.

   Then try c_1, c_2, c_3 as `2 + rand(1)/10` 100 times. Find the averages of the matrices K^{-1}.

   Compare with $c_1 = c_2 = c_3 = 2$. Any interesting conclusions?