Problem Set 4

Due: October 29, 2015 (E18-366, 1:00pm)

Name (Print):

Instructions:

- Include a printed copy of this document with your solutions.
- Organize your work, in a neat and coherent way. Work without a clear ordering will receive very little credit.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive very little credit; an incorrect answer supported by substantially correct calculations and explanations will still receive partial credit.
- Box your final answer to each problem.
- All the problems are worth the same amount (20 points/problem).

Problem 1 (A.L.): (Question 5 in section 2.4) For every connected graph, explain why the *only* solutions to $Au = 0$ are the constant vectors $u = (C, \ldots, C)$.

Problem 2 (A.L.): (Question 6 in section 2.4) The sum down the main diagonal of a matrix is the *trace*. For the complete graph (with four nodes) and the tree (with four nodes), the traces of $A^T A$ are $3 + 3 + 3 + 3 = 12$ and $1 + 2 + 2 + 1 = 6$. Why is the trace of $A^T A$ always $2m$ for any graph with m edges?

Problem 3 (T.F.):
(1) ([Section 2.7 Problem 4]) Truss D has how many rows and columns in the matrix A? Find column 1, with 8 elongations from a small displacement u_1^H. Draw a nonzero solution to $Au = 0$. Why will $A^T w = 0$ have a nonzero solution (8 bar forces in balance)?
(2) Recall that the *rank* of a matrix A is the maximal number of columns of A which are linearly independent. Deduce that the matrix A in (1) has rank 7 by showing that the matrix A' obtained from deleting row 7 and column 6 is invertible. What is the truss represented by A'? You may use MATLAB to carry out the matrix calculation, you may also work with other equivalent definitions of rank you can find.
(3) Does truss D in (1) have any mechanism?

Problem 4 (T.F.):(cf. Section 2.7 Problem 7, 10)
We will study space truss in this problem.
(1) Suppose a space truss has m bars, n_1 fixed nodes and n_2 free nodes. What is the size of

matrix A?
(2) What is the number of rigid motions in 3D? Can you describe 3D rigid motions?
(3) If a space truss is stable, what is its minimal number of fixed nodes? Why?
(4) Consider a tetrahedron truss (6 bars) with 4 nodes located at $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ respectively. Suppose that all the nodes lying on the $z=0$ plane are fixed. Write down the matrix A. Is this truss stable?

Problem 5 (S.C.):
(1) Solve $u" = -x$ with $u(0) = u(1) = 0$. then solve approximately with two hat functions and $h = \frac{1}{3}$. Where is the biggest error? Do they agree on the nodes?
(2) Let ϕ be the parabola of height 1 with zeros at $3h$ and $4h$. Use Simpson's rule to compute the area under ϕ and the area under $(\phi')^2$.
(3) Let (ϕ_i) be a set of independent trial functions such that $\phi_i(0) = \phi_i(1) = 0$. Check that the matrix $K = (\int_0^1 \phi_i{}'\phi_j{}')_{i,j}$ is positive definite.

Problem 6 (S.C.):
If Φ a finite set of independent functions, we will consider $E(\Phi) = \int_0^1 (u' - u_\Phi{}')^2$ as a measure of the error in the approximation.

Let (P) be the fixed-fixed problem $u" + 2u' + u = x + 2$ with $u(0) = u(1) = 0$

(1) What is the exact solution of (P) ? What is its weak formulation? How is the approximation u_Φ defined given a set of independent test functions Φ? (Find a matrix K and a vector F such that $u_\Phi = \sum U_i\phi_i$ where $U = (U_i)$ is the solution of $KU = F$)

We will start with linear approximations

(2) Let n be an integer and $\Phi^{(n)} = \{\phi_1, \ldots, \phi_{n-1}\}$ be the set of functions defined as follows : $\phi_i(x) = 0$ if $x \notin]\frac{i-1}{n}, \frac{i+1}{n}[$, $\phi_i(\frac{i}{n}) = 1$ and ϕ_i is linear on both $[\frac{i-1}{n}, \frac{i}{n}]$ and $[\frac{i}{n}, \frac{i+1}{n}]$. Sketch what this set of functions looks like. Why are they independent? Compute K and F in this case.

(3) Use *MATLAB* to compute U when $n = 6$. Plot u with $u_{\Phi^{(6)}}$.

(4) Explain why any piecewise linear functions on the subsegments $[\frac{i}{n}, \frac{i+1}{n}]$ lies in $H_{\Phi^{(n)}}$ and deduce the order (as a power of $1/n$) of $E(\Phi^{(n)})$.