Problem Set 1

Due: September 17, 2015

Problem 1: Calculate K_3^{-1} and $\det(K_3)$ **by hand**. What is the determinant of K_3^{-1}?

Problem 2: Carry out elimination on the 4 by 4 circulant matrix C_4 to reach an upper triangular U. How do we know that C is singular from U? The last column of U has new nonzeros. Explain why this "fill-in" happens.

Problem 3: You can multiply Ax by rows (the usual way) or by columns (more important). Do this mutplication both ways for:

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

Now, try these two ways with the matrix B_3 and the all-ones vector. Can you predict the result before doing any calculation? Check it.

Problem 4: What are the second derivative $u''(x)$ and the second difference $\Delta^2 U_n$. Use $\delta(x)$.

$$u(x) = \begin{cases} Ax & \text{if } x \le 0 \\ Bx & \text{if } x \ge 0 \end{cases} \qquad U_n = \begin{cases} An & \text{if } n \le 0 \\ Bn & \text{if } n \ge 0 \end{cases} = \begin{bmatrix} -2A \\ -A \\ 0 \\ B \\ 2B \end{bmatrix} \tag{2}$$

$u(x)$ and U are piecewise linear with a corner at 0.

Problem 5: Four samples of u can give fourth-order accuracy for du/dx at the center:

$$\frac{-u_2 + 8u_1 - 8u_{-1} + u_{-2}}{12h} = \frac{du}{dx} + bh^4 \frac{d^5u}{dx^5} + \ldots \tag{3}$$

(1) Check that this is correct for $u = 1$, $u = x^2$, and $u = x^4$.
(2) Expand u_2, u_1, u_{-1}, u_{-2} as in equation (2) [Textbook Sec 1.2]. Combine the four Taylor series to discover the coefficient b in the h^4 leading error term.

Problem 6: Show that the 3rd derivative can be appoximated by the difference:

$$\frac{d^3u_i}{dx^3} \approx \frac{u_{i+2} - 2u_{i+1} + 2u_{i-1} - u_{i-2}}{2h^3} \tag{4}$$

where h is the spacing between neighboring grid points. What is the leading error in this approximation?