Exam 3

December 3, 2015
Time limit: 60 minutes

Name (Print):

This exam contains 7 pages (including this cover page) and 3 problems. Check to see if any pages are missing.

Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use your course textbook and notes. No electronic devices or other materials are allowed.

You are required to show your work on each problem on this exam, unless otherwise specified. The following rules apply:

- **Organize your work**, in a neat and coherent way. Work without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive very little credit; an incorrect answer supported by substantially correct calculations and explanations will still receive partial credit.

- Box your final answer to each problem.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	30	
2	35	
3	35	
Total:	100	
	Total (%):	

Problem 1 (30 points):

(a) (10 points) Is the vector field $w(x, y) = (x^2 - y^2, 2xy)$ equal to the gradient of any function $u(x)$? What is the divergence of w? If $u(x, y)$ and $s(x, y)$ are a Cauchy-Riemann pair, show that $w(x, y) = (s(x, y), u(x, y))$ will be a gradient field and also have divergence zero.

(b) (10 points) Take real and imaginary parts of $f(x + iy) = (x + iy + \frac{1}{x+iy})$ to find two solutions of Laplace's equation. Write those two solutions also in polar coordinates.

(c) (10 points) Integrate each of the functions $u = 1$, $u = r\cos\theta$, $u = r^2 \cos 2\theta$ around the closed circle of radius 1 to find $\int u \, d\theta$. How could this same computation come from the Divergence Theorem?

Problem 2 (35 points):

Solve Poisson's equation with $f(x, y) = 1$ on a square of side length 1 by *finite differences* considering two *interior* nodes in the x direction and three *interior* nodes in the y direction. The boundary conditions are:

- $u = 1$ along the horizontal sides (x direction) including the corners of the square

- $u = 0$ along the vertical sides (y direction) excluding the corners of the square

** Do not compute the actual solution. Just write down the matrices involved in the calculation.

Problem 3 (35 points):

Solve Poisson's equation with $f(x, y) = 1$ on a square of side length 1 by the *finite element method* considering the non-uniform triangular grid given in the sketch below. The boundary conditions are $u = 1$ everywhere.