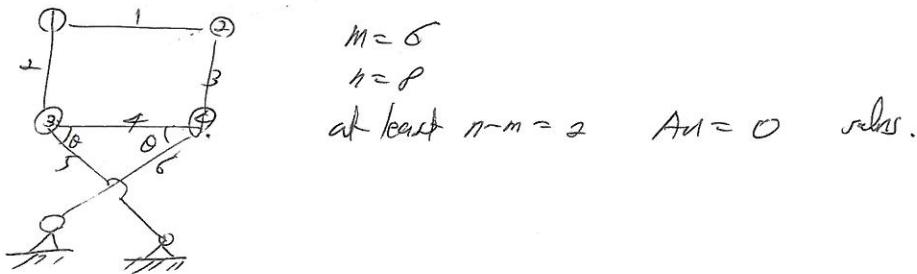
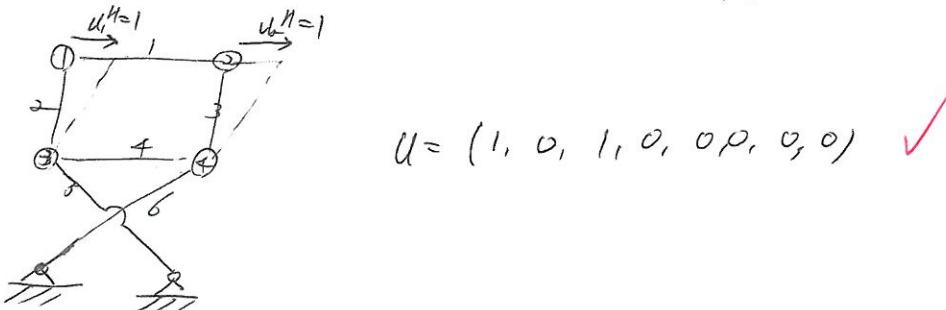


Problem Set 2.7 - Problem 1.

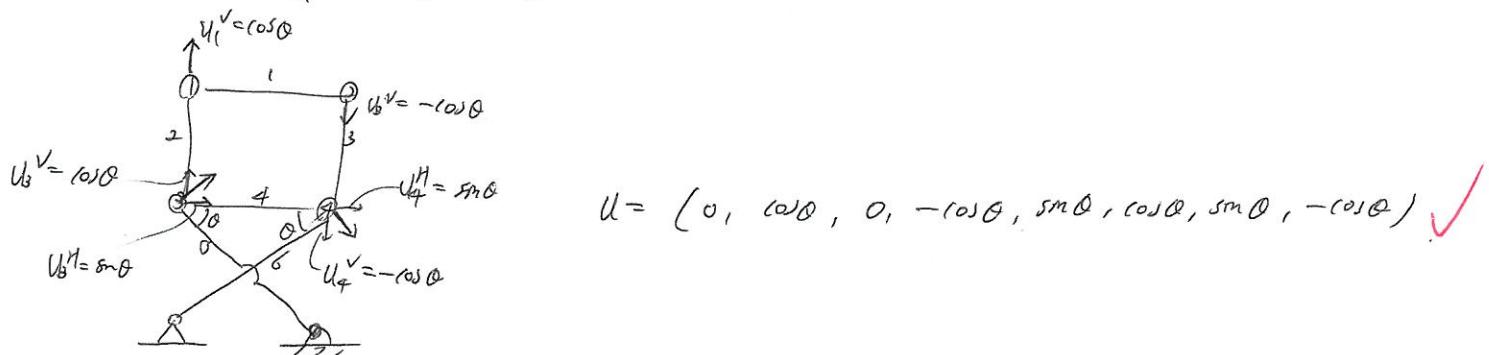


i) $u_1^H = 1, u_2^H = 1$, others all zero : ①, ② translation to the right



ii) $u_3^H = \sin\theta, u_3^V = \cos\theta, u_4^H = \sin\theta, u_4^V = -\cos\theta$: edge 5, 6 rotation clockwise

$$u_1^H = 0 \quad u_1^V = \cos\theta \quad u_2^H = 0 \quad u_2^V = -\cos\theta$$



shape $\Rightarrow [A : m \times n \Rightarrow 6 \times 6 \text{ matrix}, A^T A : n \times n \Rightarrow 6 \times 6 \text{ matrix}]$

$$C = A^T u = \begin{bmatrix} -U_1^H + U_2^H \\ U_1^V - U_3^V \\ U_4^V - U_5^V \\ -U_6^H + U_7^H \\ -U_8^H \cos\theta + U_9^V \sin\theta \\ U_9^H \cos\theta + U_8^V \sin\theta \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\theta & \sin\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \end{bmatrix} \begin{bmatrix} U_1^H \\ U_1^V \\ U_2^H \\ U_2^V \\ U_3^H \\ U_3^V \\ U_4^H \\ U_4^V \\ U_5^H \\ U_5^V \\ U_6^H \\ U_6^V \\ U_7^H \\ U_7^V \\ U_8^H \\ U_8^V \\ U_9^H \\ U_9^V \end{bmatrix}$$

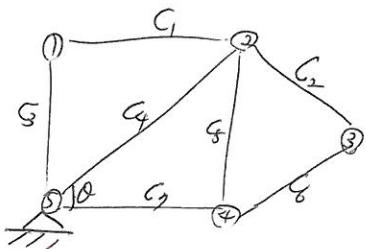
$$A \text{ 1st row} \Rightarrow [-1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0] \checkmark$$

$$A^T A \text{ 1st row} \Rightarrow [1 \ 0 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0] \checkmark$$

cf) $A^T A =$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 + \cos^2\theta & -\cos\theta\sin\theta & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -\cos\theta\sin\theta & 1 + \sin^2\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 + \cos^2\theta & \cos\theta\sin\theta & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & \cos\theta\sin\theta & 1 + \sin^2\theta & 0 \end{bmatrix}$$

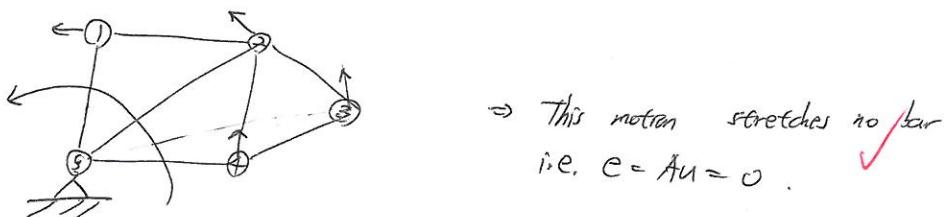
Problem Set 2.7 - Problem 2.



$$m = 7 \\ n = 2N - r = 10 - 2 = 8. \text{ unknown displacements.}$$

$$\text{at least } n-m = 1 \quad \Delta u = 0 \text{ soln.} \quad \checkmark$$

1) rotation w.r.t. node ⑤ $\Rightarrow \Delta u = 0$.



2) Adding one bar $m = 8, n = 8 \Rightarrow n - m = 0$

However, wherever the bar is installed, the rotation explained above is still possible.

\therefore Although A will be square, it will NOT be invertible. \checkmark

$$3) \text{ row 2 of } A \Rightarrow C_2 = [\text{row 2 of } A] \begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} -u_2^H/\sqrt{2} + u_2^V/\sqrt{2} & + u_3^H/\sqrt{2} & -u_3^V/\sqrt{2} \end{bmatrix}$$

$$\therefore \text{row 2 of } A = [0 \quad 0 \quad -\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}} \quad 0 \quad 0] \quad \checkmark$$

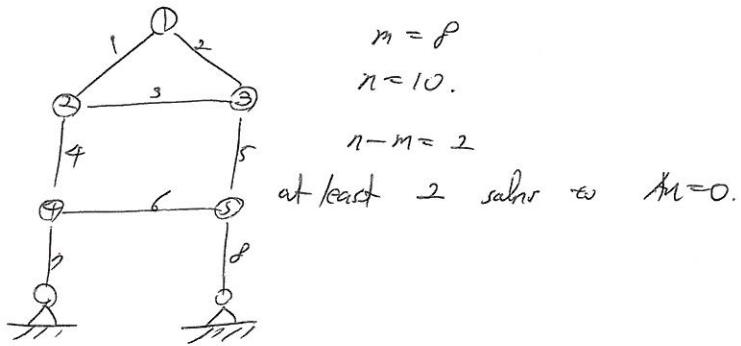
4) Third equation in $A\Delta u = f$. $f_2^H \rightarrow [0 = \text{angle between bar 4 and bar 3}]$

(assume bar 4 at a θ angle, bar 2 at a 45° angle)

$$w_1 - \frac{w_2}{\sqrt{2}} + \frac{w_4 \cos \theta}{\sqrt{2}} = f_2^H$$

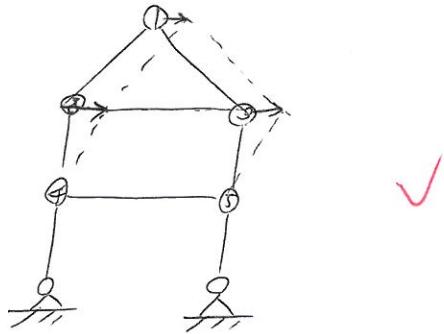
$$[\text{If } \theta = 45^\circ : \quad w_1 - \frac{w_2}{\sqrt{2}} + \frac{w_4}{\sqrt{2}} = f_2^H] \quad \checkmark$$

Problem Set 2.7 - Problem 5



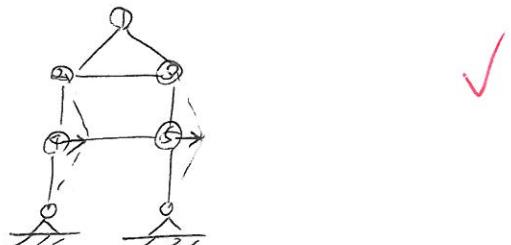
• Mechanism 1.

$$U_1^H = U_2^H = U_3^H = 1, \text{ all others zero}$$



• Mechanism 2

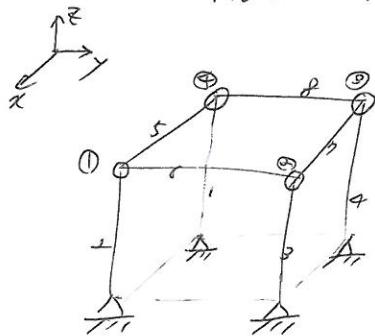
$$U_4^H = U_5^H = 1, \text{ all others zero}$$



\therefore Since solution (nonzero) to $Au=0$ exists, columns of A are not independent.

$\therefore A^T A$ is semidefinite

Problem Set 2.7 - Problem 7



$\therefore 4$ nodes on top surface
 $u^x, u^y, u^z \Rightarrow n=12$ ✓

8 edges
(4 on the side, 4 on the top surface) $\Rightarrow m=8$ ✓

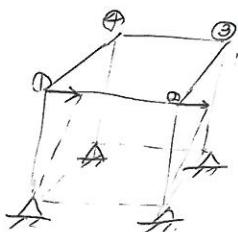
$\therefore A$ is ~~8~~ by 12 matrix ✓

2) $n-m=4$

At least 4 subs (non-zero) $\neq Au=0$.

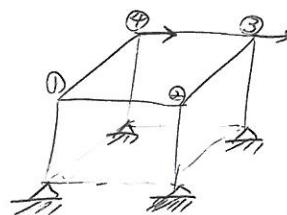
Mechanism 1.

$$u_1^x = u_2^x = 1 \text{ all others zero}$$



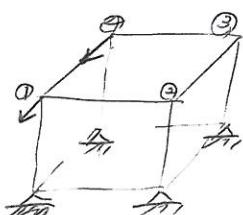
Mechanism 2

$$u_3^x = u_4^x = 1 \text{ all others zero}$$



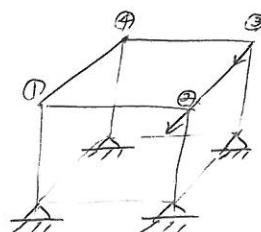
Mechanism 3

$$u_1^y = u_4^y = 1 \text{ all others zero}$$

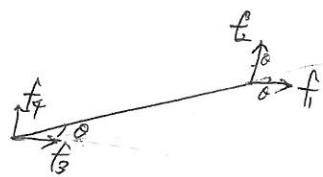
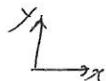


Mechanism 4

$$u_2^y = u_3^y = 1 \text{ all others zero}$$



Problem Set 2.7 - Problem 9.



$$1) \quad e = Au = u_1^H \cos \theta + u_1^V \sin \theta - u_2^H \cos \theta - u_2^V \sin \theta.$$

$$= [\cos \theta \quad \sin \theta \quad -\cos \theta \quad -\sin \theta] \begin{bmatrix} u_1^H \\ u_1^V \\ u_2^H \\ u_2^V \end{bmatrix}$$

$$\therefore A_o = [\cos \theta \quad \sin \theta \quad -\cos \theta \quad -\sin \theta] \quad \checkmark$$

$$A_o^T = \begin{bmatrix} \cos \theta \\ \sin \theta \\ -\cos \theta \\ -\sin \theta \end{bmatrix} \quad \checkmark$$

$$A_o^T C A_o = \begin{bmatrix} C \cdot \cos^2 \theta & C \cdot \cos \theta \sin \theta & -C \cdot \cos^2 \theta & -C \cdot \cos \theta \sin \theta \\ C \cdot \cos \theta \sin \theta & C \cdot \sin^2 \theta & -C \cdot \cos \theta \sin \theta & -C \cdot \sin^2 \theta \\ -C \cdot \cos \theta \sin \theta & -C \cdot \cos \theta \sin \theta & C \cdot \cos^2 \theta & C \cdot \cos \theta \sin \theta \\ -C \cdot \sin^2 \theta & -C \cdot \sin^2 \theta & C \cdot \cos \theta \sin \theta & C \cdot \sin^2 \theta \end{bmatrix} \quad \checkmark$$

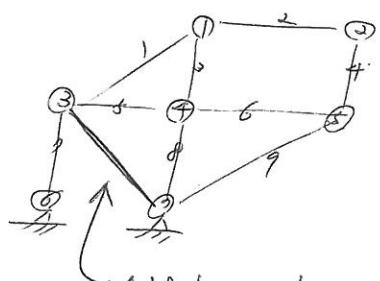
$$2) \quad A_o^T y = f$$

f should be in the form of

$$f = \alpha \begin{bmatrix} \cos \theta \\ \sin \theta \\ -\cos \theta \\ -\sin \theta \end{bmatrix} \quad \checkmark$$

where α is an arbitrary constant.

Problem Set 2.7 - Problem 11.



$$\begin{aligned}n &= 10 \\m &= 9. \\&\downarrow \text{add 1} \\m &= 10.\end{aligned}$$

✓

(∴ This bar can prevent rotation)