11 Suppose we want a one-sided approximation to $\frac{du}{dx}$ with second order accuracy:

$$\frac{ru(x) + su(x - \Delta x) + tu(x - 2\Delta x)}{\Delta x} = \frac{du}{dx} \quad \text{for} \quad u = 1, x, x^2 .$$

Substitute $u = 1, x, x^2$ to find and solve three equations for r, s, t. The corresponding difference matrix will be lower triangular. The formula is "causal."

~~Solve for u_1, u_2, u_3, u_4. Compare them to the true solution.~~

19 Construct a *centered* difference approximation using K/h^2 and $\Delta_0/2h$ to

$$-\frac{d^2u}{dx^2} + \frac{du}{dx} = 1 \quad \text{with } u(0) = 0 \text{ and } u(1) = 0 .$$

Separately use a *forward* difference $\Delta_+ U/h$ for du/dx. Notice $\Delta_0 = (\Delta_+ + \Delta_-)/2$. Solve for the centered u and uncentered U with $h = 1/5$. The true $u(x)$ is the particular solution $u = x$ plus any $A + Be^x$. Which A and B satisfy the boundary conditions? How close are u and U to $u(x)$?