Quiz 1

18.085 (Prof. Edelman)

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- Do all your work on these pages. No calculators or computers may be used. Notes and the text may be used. The point value (total is 33) of each subproblem is indicated.

1. (17 points total)

a. (5 points) Write down a second order differential equation on $-1 \leq x \leq 1$, whose solution is $u(x) = |x|$. Be sure to include the boundary conditions as well as the differential equation. Be extra sure to get the coefficients exactly correct as there may not be partial credit.

b. (8 points) What is $u_2, u_3, u_4, \ldots, u_8$ exactly in the equation

$$K_9 \begin{pmatrix} 1/2 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} ?$$

where K_9 is our special K, which we sometimes write as toeplitz([2 -1 0 0 0 0 0 0 0]).

c. (4 points) Give a mass-spring interpretation of the solution in b.

2. (16 points total) Consider fitting the data $(-1, f_1), (0, f_2), (1, f_3)$ to the curve $y = C + Dt^2$.

a. (4 points) What matrix equation $Au = f$ would we like to solve (but probably can not) for the best choice of C and D?

b. (3 points) Is $A^T A$ positive definite or only positive semi-definite?

c. (5 points) By hook or by crook, (but not with calculators or computers) decide whether $(A^T A)^{100}$ is very large, nearly 0, or something else? (Explain)

d. (4 points) Now consider fitting the same three points with the curve $y = C + Dt^2 + E(e^t + e^{-t} - 2)$. What is the new A?

Are the columns of this new matrix independent? (Explain your answer).

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