18.085 Computational Science and Engineering April 15,2010

Answers to Problem Set 8

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[3.4-1]

By assumption, we have $\Delta u = 0$ and

$$\int\int |gradu|^2 dxdy = \int u(gradu)nds$$

If $u = 0$ on the boundary, $\int\int |gradu|^2 dxdy = 0$, which implies $gradu = 0$ and thus $u = 0$ everywhere by the assumption that $u = 0$ on the boundary. If $u(gradu)n = 0$, we still have $gradu = 0$, this will implies u is a constant.

[3.4-2]

$u = x^2 + y^2 - 1$

[3.4-3]
$u = 3xy$

[3.4-12]

This map is not conformal since the angles are not preserved.

[3.4-13]
$Z = \frac{1}{Z}$: A circle of radias $\frac{1}{2}$ centered at $0, \frac{1}{2}$;

$Z = \frac{2}{Z}$ will map $|Z| < 1$ to $|Z| > 2$;

$Z = 2Z + i$.

[3.5-8,9,10] Expand the matrix by definition carefully.