18.085 Computational Science and Engineering	Feb 24,2010

Solutions to Problem Set 2

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[1.3-5]
2 and 7

[1.3-6]
15, 5 and 10

[1.3-7]
Check by direct matrix multiplication.

[1.3-13]

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Therefore, the answer is 1 and -1. Then by multiplication, we get vectors $(1, 1)$, $(2, 1)$, $(3, 2)$, $(5, 3)$, $(8, 5)$, which give us the sequence

$$1, 1, 2, 3, 5, 8, ...$$

[1.4-2]
Let $u(x) = Ax + B$ when $0 \leq x \leq a$, and $u(x) = Cx + D$, when $a \leq x \leq 1$. By the boundary conditions, $u'(0) = 0$, $u(1) = 4$. Together with the jump condition and slope change, we have

$$A = 0, B = 5 - a, C = -1, D = 5.$$

[1.4-6]
Let $u(x) = Ax + B$ when $0 \leq x \leq a$, and $u(x) = Cx + D$, when $a \leq x \leq 1$. By the periodic conditions, we have $u'(0) = u'(1)$ and $u(0) = u(1)$. By the continuity at a, we have $Aa + B = Ca + D$. But then the change in slope at $x = a$ will give us $C - A = -1$, which contradicts $C = A$ that we get from the boundary conditions.

[1.4-12]
Let $u(x) = Dx^3 + Ex^2 + Fx + G + C(x)$, where $C(x) = \frac{1}{6}x^3$ for $x > 0$. Then by the condition that $u(-1) = 0$, $u''(-1) = 0$, $u(1) = 0$ and $u''(1) = 0$, we have

$$D = -\frac{1}{12},\ E = -\frac{1}{4},\ F = 0,\ G = \frac{1}{6}.$$

[1.4-15]

$$
\begin{aligned}
\int_{-\infty}^{+\infty} g(x)\delta'(x)dx &= g(x)\delta(x)|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \delta(x)dg(x) \\
&= 0 - \int_{-\infty}^{+\infty} \delta(x)g'(x)dx \\
&= -g'(0)
\end{aligned}
$$

[1.5-2]
$\lambda = 2(1 - \cos \pi h)$

[1.5-10]
For $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$, we have

$$A = S\Lambda S^{-1} = \begin{pmatrix} 1 & \frac{1}{2^{\frac{1}{2}}} \\ 0 & \frac{1}{2^{\frac{1}{2}}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 2^{\frac{1}{2}} \end{pmatrix}.$$

Then check that $A^2 = S\Lambda^2 S^{-1}$ is direct by matrix multiplication.

Similar for $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$, we have

$$A = S\Lambda S^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}.$$

Also, we have $A^2 = S\Lambda^2 S^{-1}$.

[1.5-11]

$$A^{-1} = (S\Lambda S^{-1})^{-1} = (S^{-1})^{-1}(\Lambda)^{-1}S^{-1} = S\Lambda S^{-1}.$$

Also for A^3, we have $A^3 = S\Lambda^3 S^{-1}$.

[1.5-12]
$S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $\Lambda = \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$. Then

$$A = S\Lambda S^{-1} = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}.$$

[1.6-12]
For $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$, we have $A^T A$ positive definite because the columns of A are linearly independent.

For $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}$, we have $A^T A$ positive definite because the columns of A are linearly independent.

For $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, we have $A^T A$ not positive definite because the columns of A are linearly dependent.