SOLUTION OF PROBLEM SET ONE

Notice: This solution is only contained non-MATLAB part.

1, Problem Set 1.1.
If $i \geq j$, then $(T_n^{-1})_{ij} = i$; otherwise, $(T_n^{-1})_{ij} = j$.

5, Problem Set 1.1.
$\det(K_5) = 6$ and

$$\text{inv}(K_5) = \frac{1}{6}\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 8 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 8 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

9, Problem Set 1.1.
Verify that

$$\begin{aligned}(1,1,1,1)\cdot(2,-1,0,-1) &= 2-1-1 = 0 \\ (1,1,1,1)\cdot(-1,2,-1,0) &= -1+2-1 = 0 \\ (1,1,1,1)\cdot(0,-1,2,-1) &= -1+2-1 = 0 \\ (1,1,1,1)\cdot(-1,0,-1,2) &= -1-1+2 = 0.\end{aligned}$$

12, Problem Set 1.1.

$$U = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 3 & -2 & -1 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that the last entry of U is 0 because C is singular.

26, Problem Set 1.1.
$v = (2, -1, \text{zeroes}, -1)$.

7, Problem Set 1.2.
For $u = 1$, LHS$= 0$ and RHS$= 0$.
For $u = x^2$, LHS$= 2x$ and RHS$= 2x$.
For $u = x^4$, LHS$= 4x^3$ and RHS$= 4x^3$.
Expand u_2, u_1, u_{-1}, u_{-2}:

$$\begin{aligned}
u_2 &= u(x) + u'(x)(2h) + \frac{u''(x)}{2}(2h)^2 + \frac{u^{(3)}(x)}{3!}(2h)^3 + \frac{u^{(4)}(x)}{4!}(2h)^4 + \frac{u^{(5)}(x)}{5!}(2h)^5 + \cdots \\
u_1 &= u(x) + u'(x)h + \frac{u''(x)}{2}h^2 + \frac{u^{(3)}(x)}{3!}h^3 + \frac{u^{(4)}(x)}{4!}h^4 + \frac{u^{(5)}(x)}{5!}h^5 + \cdots \\
u_{-1} &= u(x) + u'(x)(-h) + \frac{u''(x)}{2}(-h)^2 + \frac{u^{(3)}(x)}{3!}(-h)^3 + \frac{u^{(4)}(x)}{4!}(-h)^4 + \frac{u^{(5)}(x)}{5!}(-h)^5 + \cdots \\
u_{-2} &= u(x) + u'(x)(-2h) + \frac{u''(x)}{2}(-2h)^2 + \frac{u^{(3)}(x)}{3!}(-2h)^3 + \frac{u^{(4)}(x)}{4!}(-2h)^4 + \frac{u^{(5)}(x)}{5!}(-2h)^5 + \cdots
\end{aligned}$$

And LHS= $u'(x) + \frac{1}{30}h^4u^{(5)}(x)$ and thus $b = 1/30$.

16, Problem Set 1.2.
$u = \frac{\cos(4\pi x) - x - 1}{16\pi^2}$. Use K_4 to compute $u_1, \ldots, u_4$:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 25 \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \cos(4\pi/5) \\ \cos(8\pi/5) \\ \cos(12\pi/5) \\ \cos(16\pi/5) \end{bmatrix}$$

Similarly, one can use K_8 to calculate.

18, Problem Set 1.2.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 5 \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -1 \\ -2 \\ -3 \\ -4 \end{bmatrix}$$

21, Problem Set 1.2.
$u_0 - u_1 = -hu'(0) - \frac{1}{2}h^2u''(0) + \cdots = \frac{1}{2}h^2f(0)$.