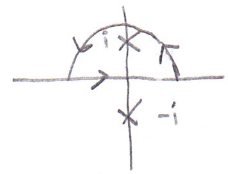
Bethany LaPenta
Hwk #4
18.075

1) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^4} = 2\pi i \,\text{Res}(i)$



transform to

$\displaystyle\int \frac{dz}{(x+i)^4(x-i)^4}$ as $R \to \infty$

quad poles at i and $-i$

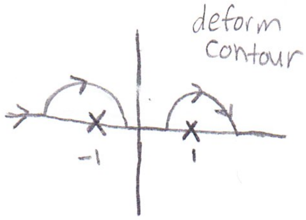
$$\text{Res}(i) = \frac{1}{3!}\frac{d^3}{d^3z}\left((z-i)^4 \cdot \frac{1}{(z-i)^4(z+i)^4}\right) = \frac{1}{6}\left(\frac{d^3}{d^3z}(z+i)^{-4}\right)$$

$$= -\frac{1}{6}\cdot 120\cdot(z+i)^{-7} \quad \text{as } z \to z_0$$

$$-20(2i)^{-7} = \frac{5}{32i} = \text{Res}(i)$$

$$2\pi i \cdot \frac{5}{32i} = \boxed{\frac{5\pi}{16}}$$

2) $\displaystyle\int_{-\infty}^{\infty} \frac{\cos^2(\frac{\pi}{2}x)}{(1-x^2)}\,dx = \int_{-\infty}^{\infty} \frac{\left(\frac{e^{\frac{i\pi x}{2}}+e^{\frac{-i\pi x}{2}}}{2}\right)^2}{1-x^2} = \frac{1}{4}\int_{-\infty}^{\infty} \frac{e^{i\pi x}+2+e^{-i\pi x}}{1-x^2}$



deform contour

split to

$\displaystyle\frac{1}{4}\oint \frac{e^{i\pi z}\,dz}{1-z} = 0 \qquad \frac{1}{4}\oint \frac{2}{1-x^2} = 0$

$$\frac{1}{4}\oint \frac{e^{-i\pi z}}{1-z^2}\,dz = \frac{1}{4}\cdot 2\pi i\cdot(\text{Res}(-1)+\text{Res}(1))$$

$$\frac{e^{-i\pi z}}{2z} \text{ @ } -1, 1 = \frac{e^{i\pi}}{-2} + \frac{e^{-i\pi}}{2} = \frac{-1}{-2} + \frac{-1}{2} = 0$$

$$\frac{1}{4}\cdot 2\pi i\cdot 0 = \boxed{0}$$