18.03 Supplementary Notes

Spring 2012

Contents

0. Preface

This packet collects notes I have produced while teaching 18.03, Ordinary Differential Equations, at MIT in 1996, 1999, 2002, 2004, 2006, 2008, 2010, and 2012. They are intended to serve several rather different purposes, supplementing but not replacing the course textbook.

In part they try to increase the focus of the course on topics and perspectives which will be found useful by engineering students, while maintaining a level of abstraction, or breadth of perspective, sufficient to bring into play the added value that a mathematical treatment offers.

For example, in this course we use complex numbers, and in particular the complex exponential function, more intensively than Edwards and Penney do, and several of the sections discuss aspects of them. This ties in with the "Exponential Response Formula," which seems to me to be a linchpin for the course. It leads directly to an understanding of amplitude and phase response curves. It has a beautiful extension covering the phenomenon of resonance. It links the elementary theory of linear differential equations with the use of Fourier series to study LTI system responses to periodic signals, and to the weight function appearing in Laplace transform techniques. It allows a direct path to the solution of sinusoidally driven LTI equations which are often solved by a form of undetermined coefficients, and to the expression of the sinusoidal solution in terms of gain and phase lag, more useful and enlightening than the expression as a linear combination of sines and cosines.

As a second example, I feel that the standard treatments of Laplace transform in ODE textbooks are wrong to sacrifice the conceptual content of the transformed function, as captured by its pole diagram, and I discuss that topic. The relationship between the modulus of the transfer function and the amplitude response curve is the conceptual core of the course. Similarly, standard treatments of generalized functions, impulse response, and convolution, typically all occur entirely within the context of the Laplace transform, whereas I try to present them as useful additions to the student's set of tools by which to represent natural events.

In fact, a further purpose of these notes is to try to uproot some aspects of standard textbook treatments which I feel are downright misleading. All textbooks give an account of beats which is mathematically artificial and nonsensical from an engineering perspective. I give a derivation of the beat envelope in general, a simple and revealing use of the complex exponential. Textbooks stress silly applications of the Wronskian, and I try to illustrate what its real utility is. Textbooks typically make the theory of first order linear equations seem quite unrelated to the second order theory; I try to present the first order theory using standard linear methods. Textbooks generally give an inconsistent treatment of the lower limit of integration in the definition of the one-sided Laplace transform, and I try at least to be consistent.

A final objective of these notes is to give introductions to a few topics which lie just beyond the limits of this course: damping ratio and logarithmic decrement; the L^2 or root mean square distance in the theory of Fourier series; the exponential expression of Fourier series; the Gibbs phenomenon; the Wronskian; a discussion of the ZSR/ZIR decomposition; the Laplace transform approach to more general systems in mechanical engineering; and an introduction to a treatment of "generalized functions," which, while artificially restrictive from a mathematical perspective, is sufficient for all engineering applications and which can be understood directly, without recourse to distributions. These essays are not formally part of the curriculum of the course, but they are written from the perspective developed in the course, and I hope that when students encounter them later on, as many will, they will think to look back to see how these topics appear from the 18.03 perspective.

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