18.03 Muddy cards Friday, March 13 (and some from previous week)

1. Questions about what will be on the exam will not be answered in text here in case people don't look here. They can be asked in review sessions in class. Professor Miller will conduct the review session for Exam 2 and will say what topics are covered. You can also look at the review sheet which is the closest to a complete list.

2. I should have asked this earlier, but what was D?

3. What is the conceptual leap between $p(D)$ and $p(r)$?

Answer to 2. The differential operator $D = \frac{d}{dt}$ is the one that take a function to its derivative.

$$Dx = \frac{d}{dt}x = x' = x'(t)$$

Therefore, if $p(D) = 2D^2 + 3D + 4$, then

$$p(D)x = 2D^2x + 3Dx + 4x = 2x'' + 3x' + 4x$$

We can use this abbreviation to talk in general about properties of differential equations.

Answer to 3. The main connection between $p(D)$ and $p(r)$ is the key formula:

$$(*) \qquad p(D)e^{rt} = p(r)e^{rt}$$

This is used in two three main ways.

a) Roots lead to solutions to the homogeneous equation. If we have $p(r_1) = 0$, then

$$p(D)e^{r_1t} = p(r_1)e^{r_1t} = 0$$

If the roots are repeated then there is another solution te^{r_1t}.

b) The roots (and repeated roots) tell us the form of a particular solution.

c) We divide the key formula (*) by $p(r)$ to see that

$$y = \frac{e^{rt}}{p(r} \quad \text{solves} \quad p(D)y = e^{rt}$$

This is the exponential response formula ERF. We will be doing more of this when we study the Laplace transform in Unit 3.

4. What was the point of the last question on PS4b?

Answer to 4. The first point was to see the large qualitative differences between homogeneous solutions in the over and underdamped cases. The second was to test the rule of thumb that the critically damped case (borderline between the oscillatory and nonoscillatory cases) is the one that brings initial conditions to equlibrium the fastest. This turns out to be roughly true, but not exactly true. This means that in any practical case, one should test either theoretically or numerically a family of initial conditions and see what will happen.

For some purposes people like to leave a little wiggle, that is, underdamping. For example it may be better to design a building that sways a little (low energy swings) in the wind. In other cases, the oscillation can't be tolerated and must be damped as promptly as possible. For example, vibrations can wreck the high resolution images from microscopes and telescopes.

The student asking this question expressed frustration about not being able to get the same answer as his fellow students. This caused him/her to waste a lot of time in front of the screen. It is too bad that you wasted a lot of time. You were not expected to get the same answers exactly — only up to a rather generous tolerance. This is as good a time to learn what is a realistic error.

5. How does the ODE change if we take into consideration the mass of the spring?

If the mass increases, it is just like dividing the damping constant, spring constant and amplitude of the input by the same larger number. That's not much of an answer. If you want to see what happens, you can look at the Forced Damped Oscillations applet. There you can vary all of the parameters: initial conditions, size of input, mass of spring, damping constant, spring constant.

6. Problem sets are too long.

My problem sets tend to be long. PS6 will be a bit shorter. You are expected to work on the course for 12 hrs per week. So if you go to recitation and to class, the problem set is suppose to take 7 hrs in addition.