Formulas to be provided in the final exam (2 pages)

Exponential response formulas

A particular solution to the equation $p(D)x = ae^{rt}$ is given by

$$\frac{ae^{rt}}{p(r)} \quad \text{if } p(r) \neq 0, \qquad \text{or} \qquad \frac{ate^{rt}}{p'(r)} \quad \text{if } p(r) = 0 \text{ but } p'(r) \neq 0.$$

Variation of parameters formula

The solutions of the linear system of equations $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{b}(t)$ are given by

$$\mathbf{x}(t) = F(t) \int F(t)^{-1}\mathbf{b}(t)\,dt$$

where $F(t)$ is any fundamental matrix for the associated homogeneous system $\mathbf{x}'(t) = A\mathbf{x}(t)$.

Coefficients of a Fourier series of period $2L$

If $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{L}\right)$, then

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right) dt \quad \text{and} \quad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\left(\frac{n\pi t}{L}\right) dt.$$

In particular, if $f(t)$ is even,

$$b_n = 0 \quad \text{and} \quad a_n = \frac{2}{L}\int_0^L f(t)\cos\left(\frac{n\pi t}{L}\right) dt;$$

if $f(t)$ is odd,

$$a_n = 0 \quad \text{and} \quad b_n = \frac{2}{L}\int_0^L f(t)\sin\left(\frac{n\pi t}{L}\right) dt.$$

Definition and properties of the Laplace transform

$f(t)$	$F(s) = \int_0^\infty f(t)e^{-st}\,dt$
$af(t) + bg(t)$	$aF(s) + bG(s)$
$e^{at}f(t)$	$F(s-a)$
$tf(t)$	$-F'(s)$

For $a \geq 0$:

$u(t-a)f(t-a)$	$e^{-as}F(s)$

Using $\mathcal{L}_+$ for a generalized derivative f':

$f'(t)$	$sF(s) - f(0^+)$

Using $\mathcal{L}_+$ for a generalized derivative f'':

$f''(t)$	$s^2F(s) - f(0^+)s - f'(0^+)$
$f(t) * g(t)$ $= \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$

Laplace transforms of some elementary functions

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$

For $a \geq 0$:

$u(t-a)$	$\frac{e^{-as}}{s}$

For $a \geq 0$ (using $\mathcal{L}_-$ for $a = 0$):

$\delta(t-a)$	e^{-as}