18.03 Spring 2009: Exam 2 Outline

The following Exponential Response Formulas will be printed on the exam:

ERF: If $p(r) \neq 0$, then

$$y = \frac{Ae^{rt}}{p(r)} \quad \text{solves} \quad p(D)y = Ae^{rt}$$

ERF′: If $p(r) = 0$, but $p'(r) \neq 0$, then

$$y = \frac{Ate^{rt}}{p'(r)} \quad \text{solves} \quad p(D)y = Ae^{rt}$$

Unit 2 concerned constant coefficient differential equations with a heavy emphasis on second-order equations. The main thing we did during the last few days was to write down ERF in the complex case:

$$z(t) = \frac{e^{i\omega t}}{p(i\omega)}$$

for the steady solution to $p(D)z = e^{i\omega t}$ and interpret what it says about steady oscillatory solutions to equations of physical significance.

1. Homogeneous solutions:

a) Find the solutions to the homogeneous equation by solving the characteristic equation and knowing the forms of the answer.

b) For oscillatory solutions, find periods, pseudoperiods, zero spacing.

c) Decide what solutions look like in the long term: tend to zero, oscillate, tend to infinity.

2. Particular solutions:

a) Find by ERF, ERF′, including complex form.

b) Find by undetermined coefficients; know the form in which to expect the solution.

3. Solve an initial value problem of the form $ax'' + bx' + cx = F$ with $x(t_0)$ and $x'(t_0)$ given. The general form will be

$$x = x_p + c_1h_1 + c_2h_2$$

Evaluate c_1 and c_2 using the initial conditions (uses methods of 1 and 2 to find h_j and x_p)

4. Interpret behavior of steady solutions: amplitude, phase lag, time lag, as a function of the input frequency ω in $mx'' + bx + kx = \cos(\omega t)$.