Student questions: February 10

Definition of "autonomous equation"?

In words: $\dot{y}$ depends on y in a way that is independent of t. In symbols: $\dot{y} = g(y)$, or $\dot{y}(t) = g(y(t))$. It's tricky: $\dot{y}$ does depend upon time—it's a function of time. But its dependence is entirely through the value of $y(t)$, not the value of t itself. $\dot{y} = 3y + 1$ is autonomous, but $\dot{y} = ty + 1$ and $\dot{y} = 3y + t$ are not.

Muddiest point: where did $k(y) = k_0(1 - y/p)$ come from?

$k(y)$ is a growth rate, measured in years^{-1} (if our unit of time is years). We'll multiply it by the current population, y, to get the rate of change of population, $\dot{y} = g(y) = k(y)y$. The logistic model posits (1) natural growth (with rate, say k_0) for small y, and (2) a maximal sustainable population (say p). So $k(0) = k_0$, and $k(p) = 0$. The simplest function $k(y)$ with these properties is the one with a straight line graph: $k(y) = k_0(1 - y/p)$. There might be reasons to use a different function $k(y)$ with these properties, but lacking them it's natural to use the simplest one. It turns out to work pretty well.

What was the ultimate answer to the Oryx question? How many licenses?

I guess I left that open. $a = .25$ (that's 250 licenses per year) led to a semi-stable equilibrium, not a good policy. So it should be less than that. How much less depends on other factors. The pset problem gets into this a little more.

Is $\dot{y} = y'$, just diff notation?

Yes. The dot reminds me that I'm thinking of the independent variable as time. The prime makes me think that the independent variable is some distance. It's just a habit of mine. You don't need to follow it.

How does the graph of $g(y)$ vs y indicate stable vs unstable?

We're talking about $\dot{y} = g(y)$ here. So if $g(c) = 0$, you have an equilibrium at $y = c$. If $g'(c) > 0$, then $g(y) > 0$ for y slightly bigger than c, and $g(y) < 0$ for y slightly less than c. So solutions just above $y = c$ are rising, and solutions just below $y = c$ are falling. That is: unstable. Reverse for stable. If $g'(c) = 0$, you can't tell what the values of $g(y)$ will be for y slightly bigger or smaller than c, so you need more information about $g(y)$ to decide what kind of equilbrium you have.

Where does "logistic" come from?

I looked this up. The equation was named by the Belgian mathematician Pierre-François Verhulst in 1838. He applied it to explain population data in Belgium, France, and a few other places. Verhulst was a professor in the Belgian Royal Military Academy. The term "logistic" meant originally "pertaining to logic," but by that time it was being used also in today's sense of provisioning military units. Perhaps Verhulst saw the entire population of Belgium as analogous to a military unit, and thought of the limits to population as a provisioning problem. [Ref: J. Kint, D. Constales, and A. Vanderbauwhede, Pierre-François Verhulst's final triumph, in *The logistic map and the route to chaos*, edited by M. Ausloos and M. Dirickx.]