Recitation 1. Feb.2  Review of double integrals.
**Task A1:** If you had trouble with double integrals in the first half of 18.02A, review them (see Simmons 20.1-4, 18.02 Notes 1). We will need them next week. (You can get help in recitation, office hours, by the tutoring service, and the Piazza discussion forum – use it!)

Lecture 2. Feb.3  Vector fields and line integrals in the plane.
**Lecture Preparation:** Read 21.1, (notes V1 optional), try to solve the following problem A2 (work due Thursday 11am in class), and post on the forum (by Thursday 9am with tag #reading2):

- What are you most confused by in the problem or reading?
- What interesting insight did you get from the reading / own thinking / other sources?
  Alternatively: Help resolve someone else’s confusion by giving an explanation. (Do not explicitly solve the problem though.)

**Problem A2.** (Using your favourite units) evaluate the work done by the force \( \vec{F} = 5y^2 \hat{i} + 3x \hat{j} \) on a particle moving from \((-1, 1)\) to \((1, 1)\)

a) if the particle moves in a straight line;

b) if the particle moves along the parabola \( y = x^2 \).

Further Homework: 4A/ 1bd, 2abc, 3abcde, 4; 4B/ 1bed, 2ab.

Lecture 3. Feb.4  Path independence and conservative fields.
**Lecture Preparation:** Read 21.2, post your questions and insights on the forum (by Friday 9am, tag #reading3), and try to solve the following problem (work due Friday 2pm in class).

**Problem A3.**

a) Is the vector field in Problem A2 conservative? Why?

b) Suppose that a particle of mass \( m \) moves along the path \( x(t) = 2t/\pi, y(t) = \cos(\pi \sin(t)) \) from \((0, 1)\) to \((1, -1)\). Calculate the work done by the constant gravitational force \( \vec{F} = -mg \hat{j} \) on this particle. (Hint: Remember potential energy and recognize that part of this information is not needed.)

Further Homework: 4C/ 1, 2, 3.

**Part A** (14 points)

Hand in problems A2,3 and the underlined problems listed above. (Now this gets graded for correctness, not just effort, so you may want to improve your earlier work. If your pre-class solution was already complete, just hand it in again. If you did not pick it up after class, it will automatically get to the grader.)

**Part B** (15 points)

**Directions:** Attempt to solve each part of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous
semesters. With each problem is the day it can be done. Write on the first page the names of all the people you consulted or with whom you collaborated and the resources you used. (Write 'none' if you did not use any collaboration or resources.)

**Problem B1.** (Tuesday, 4 points)

In general, the moment of inertia around an axis (a line) $L$ is

$$I_L = \iint_R \delta \text{dist}(\cdot, L)^2 dA,$$

where $\delta$ is the density function, and the function $(x, y) \mapsto \text{dist}((x, y), L)$ measures the distance between the point $(x, y)$ and the line $L$. For example, the usual moment of inertia around the $y$-axis (the line $x = 0$) is

$$I_{(x=0)} = \iint_R \delta x^2 dA.$$

Find the formula for the moment of inertia around the axis $x = \bar{x}$ parallel to the $y$-axis through the center of mass $(\bar{x}, \bar{y})$. Check that the parallel axis theorem holds:

$$I_{(x=0)} = I_{(x=\bar{x})} + M\bar{x}^2,$$

where $M$ is the total mass.

**Problem B2.** (Thursday, 4 points)

Consider the vector field $\vec{F} = \left( x^2y + \frac{1}{3}y^3 \right) \hat{i}$, and let $C$ be the portion of the graph $y = f(x)$ running from $(x_0, f(x_0))$ to $(x_1, f(x_1))$ (assume that $x_0 < x_1$ and that $f$ takes positive values). Show that the line integral $\int_C \vec{F} \cdot d\vec{r}$ is equal to the polar moment of inertia of the region $R$ lying below $C$ and above the $x$-axis (with density $\delta = 1$).

**Problem B3.** (Friday, 7 points: 1+2+2+2)

Consider the vector field $\vec{F}(x, y) = \frac{-y \hat{i} + x \hat{j}}{x^2 + y^2}$.

a) Show that $\vec{F}$ is the gradient of the polar angle function $\theta(x, y) = \tan^{-1}(y/x)$ defined over the right half-plane $x > 0$. (Note: this formula for $\theta$ does not make sense for $x = 0$!)

b) Suppose that $C$ is a smooth curve in the right half-plane $x > 0$ joining two points $P_0 : (x_0, y_0)$ and $P_1 : (x_1, y_1)$. Express $\int_C \vec{F} \cdot d\vec{r}$ in terms of the polar coordinates $(r_0, \theta_0)$ and $(r_1, \theta_1)$ of $P_0$ and $P_1$.

c) Compute directly from the definition the line integrals $\int_{C_1} \vec{F} \cdot d\vec{r}$ and $\int_{C_2} \vec{F} \cdot d\vec{r}$, where $C_1$ is the lower half of the unit circle running from $(1, 0)$ to $(-1, 0)$, and $C_2$ is the upper half of the unit circle, also going from $(1, 0)$ to $(-1, 0)$.

d) Since $\vec{F} = \nabla \theta$ at any point of the plane where $\vec{F}$ is defined (not just in the right half-plane $x > 0$), the vector field $\vec{F}$ ought to be conservative (path-independent). This is true in some regions, but not in others. Give an example of a region in which $\vec{F}$ is conservative, and justify your answer using the fundamental theorem of calculus for line integrals. Give another example of a region in which $\vec{F}$ is not conservative, and explain why this does not contradict the fundamental theorem.