We will begin by briefly reviewing Monsky-Washnitzer cohomology and the de Rham-Witt complex. We will then define the overconvergent de Rham-Witt complex $W^\dagger\Omega_{\overline{C}}$ for a smooth affine variety over a perfect field in characteristic p. It is a subcomplex of the de Rham-Witt complex. We show that, after tensoring with $\mathbb{Q}$, its cohomology agrees with Monsky-Washnitzer cohomology. One advantage of our construction is that it does not involve a choice of lift to characteristic zero.

To prove that the cohomology groups are the same, we define a comparison map

$$t_F : \Omega_{C^\dagger} \to W^\dagger\Omega_{\overline{C}}.$$

We cover our smooth affine $\overline{C}$ with certain special affines $\overline{B}$. For these particular affines, we decompose $W^\dagger\Omega_{\overline{B}}$ into an integral part and a fractional part and then show that the integral part is isomorphic to the Monsky-Washnitzer complex and that the fractional part is acyclic. We deduce our result for the more general $\overline{C}$ from a homotopy argument and the fact that our complex is a Zariski sheaf.