Conformal and Asymptotic Properties of Embedded Genus-g Minimal Surfaces with One End

by

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Abstract

Using the tools developed by Colding and Minicozzi in their study of the structure of embedded minimal surfaces in $\mathbb{R}^3$ [1–5], we investigate the conformal and asymptotic properties of complete, embedded minimal surfaces of finite genus and one end. We first present a more geometric proof of the uniqueness of the helicoid than the original, due to Meeks and Rosenberg [6]. That is, the only properly embedded and complete minimal disks in $\mathbb{R}^3$ are the plane and the helicoid. We then extend these techniques to show that any complete, embedded minimal surface with one end and finite topology is conformal to a once-punctured compact Riemann surface. This completes the classification of the conformal type of embedded finite topology minimal surfaces in $\mathbb{R}^3$. Moreover, we show that such s surface has Weierstrass data asymptotic to that of the helicoid, and as a consequence is asymptotic to a helicoid (in a Hausdorff sense). As such, we call such surfaces *genus-g helicoids*. In addition, we sharpen results of Colding and Minicozzi on the shapes of embedded minimal disks in $\mathbb{R}^3$, giving a more precise scale on which minimal disks with large curvature are "helicoidal". Finally, we begin to study the finer properties of the structure of genus-g helicoids, in particular showing that the space of genus-one helicoids is compact (after a suitably normalization).

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