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· P set 6, Part I · 18.01 · Fall 2009.

#5 A bucket weighing 5 lb when empty is loaded w/ 60 lb of sand. Unfortunately there is a hole in the bucket, and sand leaks out uniformly at such a rate that a third of the sand has been lost when the bucket has been lifted 10 ft. Find the work done in lifting the bucket this distance.

Assume leaks uniformly wrt height lifted. Work = Force · distance. (Force = weight of sand)

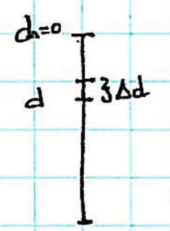
denote Weight = Weight @ height h = W(h).

$W(0) = W_0 = 5 + 60 = 65$. Losing sand at rate $-\frac{60}{3}\cdot\frac{1}{10} = -2$ (lb/ft)

⇒ know weight as function of height: $W(h) = 65 - 2h$.

$$\text{Work} = \int_0^{10} W(h)\,dh = \int_0^{10} 65 - 2h\,dh = 65h - h^2\Big|_0^{10} = 65\cdot10 - 10^2 = 650 - 100 = \boxed{550 \ \text{(lb-ft)}}$$

#6 A cable 100 ft long that weighs 4 lb/ft is hanging from a windlass. How much work is done in winding it up?



Work done to wind entire wire = Σ work done to lift each infinitesimally small piece of the wire.

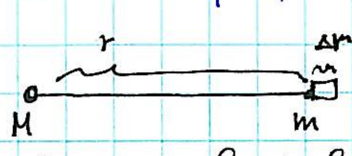
work done to lift piece d away from top = $4\cdot\Delta d\cdot d$ (weight: $4\cdot\Delta d$, distance: d)

⇒ need to $\sum 4d\Delta d$ from d = 0 to 100, take $\Delta d \to 0$ ⇒ can set up integral

$$W = \int_0^{100} 4d\,d(d) = 2d^2\Big|_0^{100} = 2(10^2)^2 = \boxed{20{,}000 \ \text{(ft-lbs)}}$$

(Sorry for poor choice of notation)

#15 If two particles of matter of masses M and m are a units apart, how much work must be done to move them twice as far apart?



Move m away from M from r = a to r = 2a.

As in example 2, $dW = F\,dr = G\frac{Mm}{r^2}\,dr$

$$\text{Work} = W = \int_{r=a}^{2a} dW = \int_{r=a}^{2a} G\frac{Mm}{r^2}\,dr = GMm\left(-\frac{1}{r}\right)\Big|_a^{2a} = GMm\left(\frac{1}{a} - \frac{1}{2a}\right) = \boxed{\frac{GMm}{2a}}$$