18.01 FALL 2009 – Problem Set 4

Due Friday 10/16/09, 1:45 pm in 2-106

Part I (10 points)

Lecture 14. Fri. Oct. 9 Mean-value theorem.
Read: 2.6 up to p. 79, Notes MVT Work: 2G-1b, 2b, 5, 6

Recitations held on Tuesday, October 13

Lecture 15. Thurs. Oct. 15 Differentials and antiderivatives.
Read: 5.2, 5.3 Work: 3A-1de, 2acegik, 3aceg

Lecture 16. Fri. Oct. 16 Differential equations; separating variables.
Read: 5.4, 8.5 Work assigned on the next problem set.

Exam 2. Tues. Oct. 20 **Exam 2** Covers Lectures 8–16. Rooms to be announced.

Part II (21 points)

Directions: Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. Beside each problem is the date on which corresponding material in class is covered.

0. (not until due date; 3 pts) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". (See full explanation on PSet1).

1. (Friday, 3 pts: 1 + 2)

(a) Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all real x? Explain.

(b) A number a is called a "fixed point for f" if $f(a) = a$. Prove that a function $f(x)$ such that $f'(x) \neq 1$ for all real x can have at most one fixed point.

2. (Friday, 10pts: 2 + 2 + 2 + 2 + 2))

a) Use the mean value property to show that if $f(0) = 0$ and $f'(x) \geq 0$, then $f(x) \geq 0$ for all $x \geq 0$.

b) Deduce from part (a) that $\ln(1 + x) \leq x$ for $x \geq 0$. Hint: Use $f(x) = x - \ln(1 + x)$.

c) Use the same method as in (b) to show $\ln(1+x) \geq x - x^2/2$ and $\ln(1+x) \leq x - x^2/2 + x^3/3$ for $x \geq 0$.

d) Find the pattern in (b) and (c) and make a general conjecture.

e) Show that $\ln(1 + x) \leq x$ for $-1 < x \leq 0$. (Use the change of variable $u = -x$.)

3. (Thursday, 2 pts) Simmons 5.3/68

4. (Thursday, 2 pts) Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.