18.01 FALL 2009 – Problem Set 1

Due Friday 9/18/08, 1:45 pm in 2-106

18.01 Supplementary Notes, Exercises and Solutions are for sale at Copy Tech in the basement of Building 11. This is where to find the exercises labeled 1A, 1B, etc. You will need these for the first day's homework.

Web site: *http://math.mit.edu/18.01* Links to syllabus, course information, and problem sets. As the semester progresses, we'll also post announcements, exam info, etc.

Part I consists of exercises given in the Supplementary Notes and solved in section S of the Notes. Of course, you should attempt to solve problems without referring to solutions in advance. These problems will be graded without many comments.

Part II consists of problems for which solutions are not given; it is worth more points. Some of these problems are longer multi-part exercises posed here because they do not fit conveniently into an exam or short-answer format. See the guidelines below (and also on the website) for which types of collaboration are acceptable, and follow them.

To encourage you to keep up with the homework as it pertains to lectures, both Part I and Part II problems are listed with the accompanying lecture in which the material will be covered.

Part I (30 points)

Notation for Listing HW: 2.1 = Section 2.1 of the Simmons book;
2.4/13 = Section 2.4 Problem 13 in Simmons
Notes G = section G of the Notes;
1A-3 = Exercise 1A-3 in Section E (Exercises) of the Notes (solved in section S)

Recitation 0. Wed. Sept. 9: Graphing functions.
Read: Notes G, sections 1-4 HW: 1A-1b, 2b, 3abe, 6b, 7b

Lecture 1. Thurs., Sept. 10: Derivative; slope, velocity, rate of change.
Read: 2.1-2.4 HW: 1B-1, 1C-1a (using definition of derivative), 3abe, 4ab (using work in 3), 5, 6 (trace axes on answer sheet)

Lecture 2. Fri. Sept. 11: Limits and continuity; some trigonometric limits
Read: 2.5 (bottom p.70-73; concentrate on examples, skip the $\epsilon - \delta$ def'n)
Read: 2.6 to p. 75; learn def'n (1) and proof "differentiable $\Longrightarrow$ continuous" at the end.
Read: Notes C HW: 1C-2, 1D-1acdfg, 3acde, 6a, 8a (remembering "diff $\Longrightarrow$ cont.")

Lecture 3. Tues. Sept. 15: Differentiation formulas: products and quotients; Derivatives of trigonometric functions.
In the following exercises, an *antiderivative* of $f(x)$ is any $F(x)$ for which $F'(x) = f(x)$.
Read: 3.1, 3.2, 3.4 HW: 1E-1ac, 2b, 3, 4b, 5ac; 1J-1e, 2

Lecture 4. Thurs. Sept. 17: Chain rule; higher derivatives.
Read: 3.3, 3.6 HW: 1F-1ab, 2, 6, 7bc; 1J- 1akm 1G-1bc, 5ab

Lecture 5. Fri. Sept. 18 Implicit differentiation; inverse functions.
Read: 3.5, Notes G section 5 HW: To be given on Problem Set 2.

Part II (43 points)

Directions and Rules: Collaboration on problem sets is encouraged, **but**

a) Attempt each part of each problem yourself. Read each portion of the problem before asking for help. If you don't understand what is being asked, ask for help interpreting the problem and then make an honest attempt to solve it.

b) Write up each problem independently. On both Part I and II exercises you are expected to write the answer in your own words.

c) **Write on your problem set whom you consulted and the sources you used.** If you fail to do so, you may be charged with plagiarism and subject to serious penalties.

d) Do not consult materials from previous semesters.

0. (3 points) Write the names of all the people you consulted or with whom you collaborated and the resources you used, or say "none" or "no consultation". This includes visits outside recitation to your recitation instructor. If you don't know a name, you must nevertheless identify the person, as in, "tutor in Room 2-106," or "the student next to me in recitation." Optional: note which of these people or resources, if any, were particularly helpful to you.

This "Problem 0" will be assigned with every problem set. Its purpose is to make sure that you acknowledge (to yourself as well as others) what kind of help you require and to encourage you to pay attention to how you learn best (with a tutor, in a group, alone). It will also greatly help us to know what resources you find useful.

1. (Wed, 3 pts) Express $(x-1)/(x+1)$ as the sum of an even and an odd function. (Simplify as much as possible.)

2. (Thurs, 4 pts) a) Use the following table of approximate square roots to give an approximate value of $\sqrt{102}$. You should begin by finding an approximate answer for the tangent line to $\sqrt{x}$ at $x = 100$, and using your answer to compute an approximate value of $\sqrt{102}$.

x	$\sqrt{x}$
100	10
101	10.049875
100.1	10.004998
100.01	10.000499

b) Use your tangent line from part (a) to give an estimate for $\sqrt{400}$. Is your approximation larger or smaller than the correct answer? Draw a picture to illustrate your answer. (Later, we'll use calculus to give a precise method for such approximations and associated error estimates.)

3. (Thurs, 4 pts) A 15 ft. tall street lamp is placed at the very top of a hill. Suppose the 200 ft. tall hill has an outline shaped like a parabola, and so approximated by the equation $y = 200 - x^2$. What is the lowest height on the hill y at which you can read a book on a cloudy night?

4. (Thurs, 6 pts) 3.1/21 (in Simmons, on parabolic mirrors)

5. (Thurs, 4 pts: 2 + 2)

a) A water cooler is leaking so that its volume at time t in minutes is $(10-t)^2/5$ liters. Find the average rate at which water drains during the first 5 minutes.

b) At what rate is the water flowing out 5 minutes after the tank begins to drain?

6. (Thurs, 5 pts) A prospective student sits in on the last 5 minutes of Thursday's lecture, and sees on the board that the slope of the tangent line can be defined by

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h},$$

but doesn't understand what this formula, or its parts, signify. Write a paragraph (or two) explaining the parts of this formula to this student.

7. (Friday, 8 pts: 1 + 1 + 1 + 1 + 1 + 1 + 2) Simmons 2.5/19d (put $u = 1/x$), 19f, 19g, 20c, 20g (show work); 22a (needs a calculator), 22b (see the proof on page 73).

8. (Tuesday, 6 pts: 2 + 2 + 2)

a) If u, v and w are differentiable functions, find the formula for the derivative of their product, $D(uvw)$.

b) Generalize your work in part (a) by guessing the formula for $D(u_1u_2\cdots u_n)$—the derivative of the product of n differentiable functions.

c) Then prove your formula by mathematical induction (i.e., prove its truth for the product of $n+1$ functions, assuming its truth for the product of n functions).