Aside: compute

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{0-(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos x}\,\frac{1}{\cos x} = \tan x \sec x = \sec x \tan x$$

$$\frac{d}{dx}\tan x = \frac{\overbrace{\cos^2 x - \sin x(-\sin x)}^{1}}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Worksheet 4: More Derivatives

18.01 Fall 2009

Problem 1. Compute the following derivatives:

a) $\frac{d}{dx}\sin(1/(4x+2)) = \cos\left(\frac{1}{4x+2}\right)\left(-\frac{1}{(4x+2)^2}\right)(4) = -\frac{4\cos(1/(4x+2))}{(4x+2)^2} = -\frac{\cos(1/(4x+2))}{(2x+1)^2}$

b) $\frac{d}{dx}\sec^2(x+\sec x) = 2\sec(x+\sec x)\left(\sec(x+\sec x)\tan(x+\sec x)\right)\cdot(1+\sec x\tan x)$

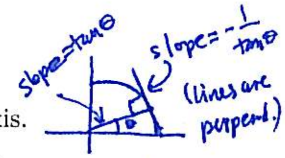
$= 2(1+\sec x\tan x)\left(\sec^2(x+\sec x)\tan(x+\sec x)\right).$

c) $\frac{d}{dx}\tan(\sin x) = \sec^2(\sin x)\cdot\cos x = \cos x\,\sec^2(\sin x).$

Problem 2. Suppose $f(3) = 3$, $f'(3) = 5$, $g(3) = 4$, $g'(3) = 5$.
What is $(f^2(x)g(x))'|_{x=3}$?

$= \left(2f(x)f'(x)g(x) + f^2(x)g'(x)\right)\Big|_{x=3} = 2f(3)f'(3)g(3) + f^2(3)g'(3) =$

$= 2\cdot3\cdot5\cdot4 + 3^2\cdot5 = 120 + 45 = 165$



Problem 3. a) Determine the tangent line to the unit circle at $\theta = \pi/6$, measured from the positive x-axis. First compute the slope by implicit differentiation, and then sanity-check your answer using trigonometry.

$x^2+y^2=1$ @ $(\cos\pi/6, \sin\pi/6) = (\sqrt{3}/2, 1/2)$

$2x + 2y\,dy/dx = 0 \Rightarrow dy/dx = -x/y = -\frac{\sqrt{3}/2}{1/2} = -\sqrt{3}.$ || Slope $= -1/\tan\theta = -1/(1/\sqrt{3}) = -\sqrt{3}$ Yes!

$\Rightarrow y - 1/2 = -\sqrt{3}(x - \sqrt{3}/2)$

$\boxed{y = -\sqrt{3}x + 2}$

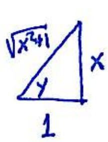
b) What is dy/dx on the curve $x^{3/2} + y^{5/2} = 1$ at the point $(2^{-2/3}, 2^{-2/5})$?

$\frac{3}{2}x^{1/2} + \frac{5}{2}y^{3/2}\frac{dy}{dx} = 0 \qquad \frac{dy}{dx}\Big|_{\text{given pt}} = -\frac{3x^{1/2}}{5y^{3/2}}\Big|_{\text{given pt}} = -\frac{3(2)^{-1/3}}{5(2)^{-3/5}} = -\frac{3}{5}(2)^{4/15}$

Problem 4. Find the derivative of $\arctan(x)$ (a.k.a. $\tan^{-1}(x)$)

$y = \arctan x \Rightarrow \tan y = x$. Take $\frac{d}{dx}$ of both sides

$\sec^2 y\,\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{x^2+1}$



$\sec^2 y = (\sqrt{x^2+1})^2 = x^2+1.$

$\boxed{\frac{dy}{dx} = \frac{1}{x^2+1}}$

Problem 5. $d^{97}(\sin x)/dx^{97} = ?$

$\left\{\sin x \xrightarrow{1} \cos x \xrightarrow{2} -\sin x \xrightarrow{3} -\cos x \xrightarrow{4} \sin x\right.$

$97 \bmod 2 = 1 \Rightarrow (\pm1)\cdot\cos x$

Sign?

$97 \bmod 4 = 1 \quad (96 \bmod 4 = 0)$

$\Rightarrow$ sign $= +1$

$\Rightarrow \boxed{\frac{d^{97}}{dx^{97}}\sin x = \cos x}$