Worksheet 2: Intro to Derivatives

18.01 Fall 2009

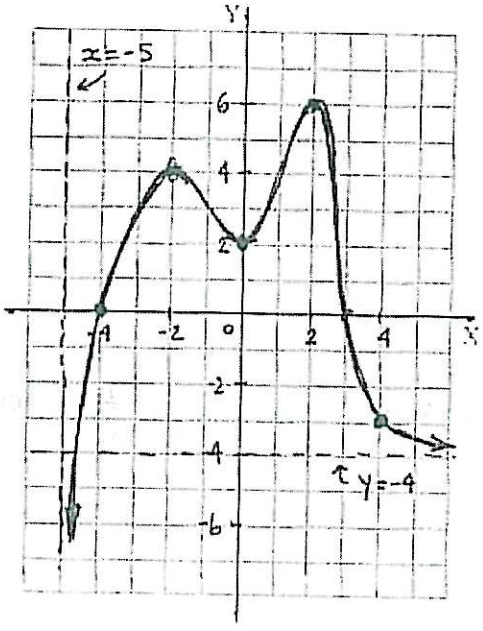
Problem 1. Let

$$y = \frac{1}{1-x}$$

Use the definition of the derivative to find the formula for dy/dx

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{1-(x+h)} - \frac{1}{1-x}}{h} = \lim_{h\to 0} \frac{1-x-(1-(x+h))}{h(1-(x+h))(1-x)} = \lim_{h\to 0} \frac{h}{h(1-(x+h))(1-x)} = \frac{1}{(1-x)^2}$$

Problem 2. Suppose the following graph from the first worksheet is the graph of the derivative $g'(x)$ of a certain function $y = g(x)$ defined on the interval $x > -5$.



a) What is the formula for the tangent line to $y = g(x)$ at the point (2,3)? How do the graphs of $y = g(x)$ and this tangent line look like relative to each other in a small neighborhood of $x = 2$?

From graph above, $g'(2) = 6$ ⇒ tangent line formula/eq.: $y = 6x + b$. Solve for b: $3 = 6\cdot 2 + b \Rightarrow b = 3 - 12 = -9$.

$y = 6x - 9$

b) What is the behavior of $y = g(x)$ as $x \to -5^+$ and $x \to \infty$?

as $x \to -5^+$, $g(x) \to$ ∞ (or finite or ∞) approaches in a spike (local)

(as $x \to \infty$, $g(x) \to -4x$) (so tends to $-\infty$)

c) At which values of x does $g(x)$ attain maximum and minimum values? Where, if anywhere, is it increasing fastest? decreasing fastest?

No global max/min. local max : $x = 3$ local min : $x = -4$ increasing fastest : $x = 2$ decreasing fastest : nowhere (not attained)

d) Suppose you could find out the value of $g(x)$ at any *one* value of x. Which point could you ask for to determine whether $g(x)$ is only negatively-valued for positive values of x?

(*) some (other) antiderivative - so that do not have pt from (a)...

ask for value at local max - @ $x = 3$.

d*) Looking ahead/discussion: what if you could only find out the value *at* $x = 0$. What are some cases in which you could still answer the above question, using only that value and this relatively sketchy graph?

Would know answer if val. at $x=0$ were either ≥ a for some not too negative (or positive) a, or (for answer - "no")
were sufficiently negative (for answer "yes") Basically, have a bound on increase in $g(x)$ from
that you would know the answer. $x=0$ to $x=3$. By a very nominal / first approximation
looking ahead { Know this (looking ahead) by counting boxes below and above graph e.g. } $6 \le \Delta g \le 18$ so if $x \le -6 \to$ "no"

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