

Practice Problems

- (1) What are the largest and smallest possible values taken of the product of three distinct numbers, spaced such that the middle number is distance one away from the other two, if the middle number is in the interval $[-2, 2]$?
- (2) By sketching the graph of $y = f(x) = x^3 - 3x - 5$, show that it has only one real root.
- (3) How many solutions to the equation $\sqrt{x} = x$? Why?
- (4) Find the triangle of smallest area in the half-plane to the right of the y -axis whose three sides respectively are segments of the x -axis, the line $y = x$, and a line through $(2, 1)$.
- (5) Calculate $\sqrt[3]{10}$ to six decimal places of accuracy.
- (6) A boat is being pulled into a dock by means of a rope with one end tied to the bow of the boat and the other end passing through a ring attached to the dock at a point five ft higher than the bow of the boat. If the rope is being pulled in at a rate of 4 ft/s, how fast is the boat moving through the water when 13 ft of rope are out?
- (7) A cylindrical tank without a top is to have a specified volume. If the cost of the material used for the bottom is three times the cost of that used for the curved lateral part, find the ratio of the height to the diameter of the base for which the cost is the least.
- (8) Show that Newton's method applied to the function $f(x) = \sqrt[3]{x}$ leads to $x_2 = -2x_1$ and is therefore useless for finding where $f(x) = 0$. Sketch the situation.
- (9) Draw a reasonably good sketch of $y = \sqrt{x}$ and mark the point on the graph that seems the closest to $(1/2, 0)$. Then calculate the coordinates of this point.
- (10) A point moves around the circle $x^2 + y^2 = a^2$ in such a way that $\frac{dx}{dt} = -y$. Find $\frac{dy}{dt}$ and decide whether the direction of the motion is clockwise or counterclockwise.
- (11) What is the smallest value of the constant a for which the inequality $ax + 1/x \geq 2\sqrt{2}$ is true for all $x > 0$?