Efficient low-power terahertz generation via on-chip triply-resonant nonlinear frequency mixing

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In this letter, we show theoretically how the light-confining properties of triply-resonant photonic resonators can be tailored to enable dramatic enhancements of the conversion efficiency of terahertz (THz) generation via nonlinear frequency down-conversion processes. Using detailed numerical calculations, we predict that this approach can be used to reduce up to three orders of magnitude the pump powers required to reach quantum-limited conversion efficiency of THz generation in conventional nonlinear optical material systems. Furthermore, we propose a realistic design readily accessible experimentally, both for fabrication and demonstration of optimal THz conversion efficiency at sub-W power levels. © *2010 American Institute of Physics*. [doi:10.1063/1.3359429]

Achieving efficient terahertz (THz) generation using compact turn-key sources operating at room temperature and modest power levels represents one of the critical challenges that must be overcome to realize truly practical applications based on THz.[1] Up to now, the most efficient approaches to THz generation at room temperature—relying mainly on optical rectification schemes—require intricate phase-matching set-ups and powerful lasers.[2,3] Recently, different approaches to resonant enhancement of difference-frequency nonlinear coupling processes, including those involving a final frequency in the THz regime, have been proposed.[4,5] However, none of these previous approaches, as they stand now, allow reaching conversion efficiencies close to the so-called quantum-limit[6] in a compact device operating at room-temperature. In this letter, we present a scheme that enables enhancement of THz power generation via second-order nonlinear frequency down-conversion by up to three orders of magnitude compared to conventional nonresonant approaches. By using a combination of accurate numerical simulations and a rigorous coupled-mode theory, we show how the unique properties of photonic microresonators to confine light in subwavelength volumes for many optical periods enable the implementation of highly-efficient compact on-chip continuous-wave (cw) THz sources operating at room temperature and pumped by sub-W pulses, which could contribute to the practical realization of efficient THz sources that are turn-key and low cost.

Our approach is motivated by the following physical picture of an arbitrary cavity-enhanced second-order nonlinear difference-frequency generation process. Consider a resonant nonlinear electromagnetic (EM) cavity characterized by a certain second-order nonlinear susceptibility tensor $\chi^{(2)}_{ijk}(\mathbf{r})$ (subindices $\{i,j,k\}$ stand for the Cartesian components $\{x,y,z\}$, respectively). Imagine further that the cavity is designed to confine, both spatially and temporally, the frequency difference ω_T, but it is otherwise transparent for both the pump and idler frequencies (denoted by ω_1 and ω_2, respectively, defined so $\omega_T=\omega_1-\omega_2$). In such a system, the temporal variation in the nonlinear polarization vector $\mathbf{P}^{\mathrm{NL}}(\mathbf{r},t)$, induced in the system by the pump and idler electric fields [$\mathbf{E}_1(\mathbf{r},t)$ and $\mathbf{E}_2(\mathbf{r},t)$, respectively], yields a current distribution, $\mathbf{J}_T(\mathbf{r},t)=\partial\mathbf{P}^{\mathrm{NL}}(\mathbf{r},t)/\partial t$, which emits radiation at ω_T inside the cavity. The power radiated by $\mathbf{J}_T(\mathbf{r},t)$, and, therefore, the overall efficiency of the nonlinear frequency-mixing process, is strongly enhanced as a result of the significant modification of the EM density modes induced by the cavity in the medium in which the current $\mathbf{J}_T(\mathbf{r},t)$ is embedded (see for instance, Ref. 7); much in the same way as the spontaneous emission rate of a quantum emitter is enhanced when it is placed inside a resonant cavity, the so-called Purcell enhancement.[8] In fact, noticing that the power radiated by $\mathbf{J}_T(\mathbf{r},t)$ inside the cavity is given by $-(1/2)\mathrm{Re}[\int_{V_{\mathrm{NL}}} d\mathbf{r}\mathbf{J}_T(\mathbf{r},t)\mathbf{E}^*_T(\mathbf{r},t)]$ (where V_{NL} denotes the volume of the nonlinear cavity and $\mathbf{E}_T(\mathbf{r},t)$ corresponds to the cavity resonant mode at ω_T; normalized so that $U_T=(1/2)\int_{V_{\mathrm{NL}}}\epsilon_0 n_T^2|\mathbf{E}(\mathbf{r},t)|^2$ is the EM energy stored in that mode, with n_T being the refractive index of the cavity at ω_T), and assuming that all the radiated power at ω_T is collected by means of a waveguide coupled evanescently to the cavity, one finds that a simple coupled-mode theory approach to this problem[9] yields the following expression for the power P_T emitted at ω_T

$$P_T=\left(\frac{4\pi c_0 n_T}{\epsilon_0\lambda_T^4}\right)\left(\frac{Q_T}{Q_{T,s}}\right)\left(\frac{Q_T}{\tilde{V}_T}\right)|a_1(t)|^2|a_2(t)|^2|\beta_{\mathrm{eff}}|^2, \qquad (1)$$

where Q_T and $Q_{T,s}$ stand for the total quality factor and the external quality factor (i.e., the one governing the decay into the waveguide) of the resonator at ω_T, respectively. λ_T denote the resonant wavelength corresponding to ω_T, while $\tilde{V}_T=V_T/(\lambda_T/n_T)^3$, where V_T is the effective modal volume of the resonant mode at ω_T. $a_1(t)$, and $a_2(t)$ are the modal amplitudes of the pump and idler electric fields inside the cavity (i.e., we define $\mathbf{E}_{1,2}(\mathbf{r},t)=a_{1,2}(t)\tilde{\mathbf{E}}_{\mathbf{1,2}}(\mathbf{r})$, where $\tilde{\mathbf{E}}_{\mathbf{1,2}}(\mathbf{r})$ represents the spatial mode profile inside the cavity at $\omega_{1,2}$, normalized so that $|a_{1,2}(t)|^2$ is the EM energy stored in the cavity at the corresponding resonant frequency $\omega_{1,2}$). Finally, the

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parameter $\beta_{\rm eff}$ governs the nonlinear coupling strength among the EM fields involved in the nonlinear difference-frequency mixing; essentially, it corresponds to the overlapping integral, weighted with the second-order nonlinear susceptibility, of the three electric fields involved in the considered nonlinear process (see Ref. 10 for details on this magnitude).

From Eq. (1), the enhancement of the power radiated by $\mathbf{J}_T(\mathbf{r},t)$ inside the cavity is apparent through the factor $Q_T/\tilde{V}_T$. If we now assume that, in addition to the confinement at the difference frequency ω_T, the cavity is also designed to trap light at ω_1 and ω_2 (forming a triply-resonant system, i.e., a configuration in which pump, idler, and signal fields have resonant cavities associated with them), another enhancement factor proportional to the product $Q_1 Q_2$ (where Q_1 and Q_2 are the quality factors of the cavity at ω_1 and ω_2, respectively) is introduced in the efficiency of the nonlinear conversion process, simply due to the recirculation of the pump and idler powers inside the cavity. Importantly, noticing that in this case $|a_1(t)|^2=(4Q_1/\omega_1)P_{1\rm in}$ (where $P_{1\rm in}$ is the input power at ω_1), and that a similar expression holds for $|a_2(t)|^2$, from Eq. (1) one can show that (using realistic parameters, and keeping fixed the values of the pump power and area of interaction) the approach proposed in this work can introduce an enhancement factor for P_T as large as 10^3 with respect to the value of P_T one would obtain in a difference-frequency generation process taking place, for instance, in a conventional phase-matched waveguide system.[11]

In order to explore the extent to which this concept could contribute to solve the current lack of efficient THz sources operating at room temperature, we illustrate its implementation in a specific structure based on a triply-resonant nonlinear configuration. As shown in the schematic displayed in Fig. 1(a), in the particular configuration considered in this work, the THz field is confined by means of a THz-scale photonic crystal (PhC) cavity, whereas the pump and idler field are resonantly confined using two whispering gallery modes (WGMs) supported by a dielectric ring resonator. More specifically, the power carried by two near-infrared (NIR) beams of wavelengths λ_1 and λ_2 (playing the role of pump and idler beams, respectively, their corresponding power being $P_{1\rm in}$ and $P_{2\rm in}$) is coupled, by means of an index-guided waveguide, to two high-order WGM, characterized by angular momenta m_1 and m_2, respectively. The ring resonator also acts as a dipole-like defect at λ_T, when embedded in an otherwise perfectly periodic THz-wavelength scale PhC formed by a square lattice of dielectric rods [see the corresponding electric field profile in Fig. 1(b)]. Thus, the $\chi^{(2)}$ nonlinear frequency down-conversion interaction that takes place between the two NIR WGM's circulating inside the ring resonator yields a current distribution that radiates inside the PhC cavity at the frequency difference $\omega_T=\omega_1-\omega_2$; as mentioned, the rate at which the radiation is emitted is strongly enhanced by the PhC environment in which the ring resonator is embedded. In order to extract efficiently the THz output power (P_T) from the PhC cavity, we introduce into the system a PhC waveguide created by reducing the radius of a row of rods [see Figs. 1(a) and 1(b)]. In addition, in order to break the degeneracy existing between the x- and y-oriented dipole defect modes, and thus, further increase the efficiency of our approach, the radius of two of the nearest

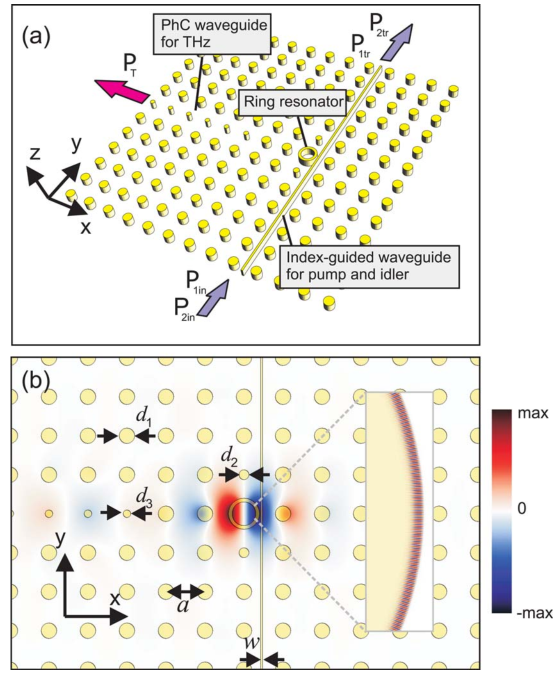

FIG. 1. (Color online) (a) Schematic of the triply-resonant nonlinear photonic structure analyzed in the text. $P_{1\rm in}$ and $P_{2\rm in}$ denote the input powers at the pump and idler frequencies, respectively; whereas $P_{1\rm tr}$ and $P_{2\rm tr}$ represent the corresponding transmitted powers through the structure. P_T stands for the THz output power. (b) Main panel: Electric field profile (E_z) corresponding to the resonant mode appearing at 1 THz in the structure shown in (a). The value of the different geometrical parameters displayed in this figure are $a=102$ μm, $d_1=40.8$ μm, $d_2=25.1$ μm, $d_3=18.8$ μm, and $w=0.8$ μm. Inset displays an enlarged view of the electric field profile (E_x) corresponding to a whispering gallery with $m=572$ circulating inside the dielectric ring shown in the main figure. The internal and external radii defining the ring resonator are 30.5 μm and 40.1 μm, respectively. Shaded areas in both the main and inset figures represent GaAs regions, while white areas represent air.

neighbors rods of the ring resonator is reduced with respect to the radius of other rods in the PhC. The whole configuration permits having a large value for factor $(Q_T/\tilde{V}_T)$, along with a high-Q resonant confinement also for the pump and idler frequencies.

Figure 1(b) shows the structure that results from optimizing the geometrical parameters of the system for efficient generation at 1 THz, along with the corresponding electric field profiles, as obtained from finite-difference time-domain (FDTD) (Ref. 12) simulations. In these calculations we have assumed a pump beam of wavelength $\lambda_1=1550$ nm, an idler beam with $\lambda_2=1542$ nm, and that the structure is implemented in GaAs [in which the relevant component of the nonlinear susceptibility tensor is $d_{14}=274$ pm/V (Ref. 13)]. For GaAs, and for the above cited values for λ_1 and λ_2, we have found that the strength of the nonlinear coupling coefficient that governs the energy transfer between the pump, idler and THz fields is maximized if the structure is designed to support two WGM with $m_1=572$ and $m_2=575$ at λ_1 and λ_2, respectively [see inbox of Fig. 1(b)], and a dipole defect mode in the THz-scale PhC.

We emphasize that in conventional phase-matching schemes, the overall efficiency of a difference-frequency generation (DFG) process relies entirely on finding a suitable nonlinear material whose dispersion relation permits fulfilling simultaneously, for the frequency range of interest, both the frequency-matching and the phase-matching conditions[6]

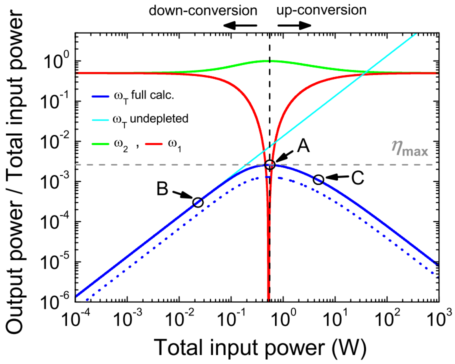

FIG. 2. (Color online) Ratio between the total output power emitted by the system at 1 THz and the total input power at the NIR pump and idler frequencies. The results for the three frequencies involved in the considered nonlinear down-conversion process are displayed (ω_1, ω_2, and ω_T correspond to the pump, idler, and final THz frequencies, respectively). Horizontal dashed line displays the maximum possible conversion efficiency η_{max} given by the Manley–Rowe quantum limit. Dotted line displays the effect of linear absorption losses on the conversion efficiency.

(or alternatively, on finding some physical mechanism, such as quasiphase-matching, that permits matching of the different fields involved in the nonlinear process). However, in the approach introduced here, the dispersion relation corresponding to the final frequency ω_T is different from that corresponding to ω_1 and ω_2 and, importantly, it can be tailored almost at will simply by modifying the geometrical parameters that define the THz-scale PhC. This introduces a general and versatile route to phase-matching that does not depend exclusively on the intrinsic properties of naturally existing nonlinear optical materials, which could be particularly relevant in those systems in which the canonical phase-matching condition cannot be fulfilled.

To compute accurately the nonlinear optical dynamics of the structure sketched in Fig. 1(a), in both the undepleted and depleted regimes, we have applied a temporal coupled-mode theory[14] (see details in Refs. 15 and 10). Figure 2 summarizes the results obtained in the cw regime. In these calculations we have assumed that $P_{1\text{in}}=P_{2\text{in}}$ and quality factors $Q_1=Q_2=3.5\times10^5$ and $Q_T=10^3$. These values for Q are compatible with both the absorption coefficient of GaAs at 1 THz (the linear absorption rate of GaAs corresponds to a Q factor 1.5×10^3)[16] and the experimental values for the quality factors obtained in similar configurations for the considered ring resonator and also for the PhC cavity.[17,18] As shown in Fig. 2, for values of the total input power ($P_{\text{tot,in}}=P_{\text{tot,in}}+P_{2\text{in}}$) larger than 0.07 W the conversion efficiency (defined here as ratio $P_T/P_{\text{tot,in}}$) starts departing from the conversion efficiency predicted by the undepleted approximation, eventually reaching the maximum value predicted by the Manley–Rowe relation.[6] As clearly shown in Fig. 2, at the critical value of the total input power at which this maximum conversion efficiency is reached ($P^c_{\text{tot,in}}=0.54$ W) the pump power that is coupled to the ring resonator is completely down-converted inside the system to power at THz and idler frequencies, giving rise to a sharp minimum in P_{1tr} and a maximum in P_{2tr}. This represents a dramatic reduction of about three orders of magnitude in $P_{\text{tot,in}}$ with respect to the most efficient schemes for THz generation in nonlinear crystals reported up to date.[2,3] Furthermore, we emphasize that, in addition to powerful lasers, current efficient schemes for THz generation require intricate phase-matching set-ups, whereas in the system introduced in this manuscript the maximum theoretically possible efficiency can be achieved in an integrated structure having a total area of approximately 1 mm^2 and using <1 W power levels, which are readily accessible with compact turn-key sources. Note that since $P_0\propto 1/Q_1Q_2Q_T$, the value of $P^c_{\text{tot,in}}$ can be adjusted just by varying the product $Q_1Q_2Q_T$. We also point out that the net effect of the absorption losses in the conversion efficiency consists simply in downscaling the results obtained in the lossless case by a factor $Q_T/Q_{T,s}$ (see dotted line in Fig. 2).

In conclusion, we have shown the dramatic enhancement of the conversion efficiency of general difference-frequency downconversion processes enabled by triply-resonant photonic resonators. By means of detailed numerical simulations, we have illustrated the relevance of the proposed scheme by demonstrating that in the cw regime the pump powers required to reach quantum-limited conversion efficiency can be reduced dramatically with respect to the conventional approaches for THz generation employed up to date. We believe these results could enable a broader use of THz sources.

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