18.175: Lecture 20

Infinite divisibility and Lévy processes

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Outline

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Infinite divisibility

Higher dimensional CFs and CLTs

Infinitely divisible laws

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- More general constructions are possible via Lévy Khintchine representation.

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- For example, given a random vector (X, Y, Z), we can define $\phi(a, b, c) = Ee^{i(aX+bY+cZ)}$.
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- The inversion theorems and continuity theorems that apply here are essentially the same as in the one-dimensional case.