EC2 SUPPLEMENT: ORIGINAL EDITION OF 1999

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Here I will maintain supplementary material for *Enumerative Combinatorics*, volume 2 (original edition of 1999). This will include errata, updated references, and new material. I will be continually updating this supplement.

NOTE. References to math.CO refer to the combinatorics section of the Mathematics Archive at arxiv.org/list/math.CO/recent. A front end site for math.CO is front.math.ucdavis.edu/math.CO.

- p. 2, Example 5.1.2. Interchange $\cap$ and $\cup$ on line 2.
- p. 6, line 10. Change situtations to situations.
- p. 8, line 6. The first Π should be $\mathbf{\Pi}$.
- p. 11, line 3. Change $E_c(n)$ to $E_c(x)$.
- p. 18, line 3. Change $(n)_2$ to $n(n-2)$.
- p. 20, line 9. Change $Z(\mathfrak{S}_n)$ to $\tilde{Z}(\mathfrak{S}_n)$.
- p. 24, line 4 (after figure). Change $\lim_{n\to\infty}$ to $\lim_{k\to\infty}$.
- p. 25, line 5. Change $\subseteq$ to $\in$.
- p. 33, line 5–. Change $\mathrm{ord}(\tau_k)$ to $\mathrm{ord}(\tau_j)$.
- p. 34, Lemma 5.3.9. Delete the first sentence, viz., "Let $w \in \mathcal{A}^*$."
- p. 35, line 10. Change $w \in \mathcal{B}^*$ to $w \in \mathcal{B}^*_{\mathbf{r}}$.
- p. 35, line 8–. Change $\mathbb{A}$ to $\mathcal{A}$.
- p. 36, lines 15–16. Change "beginning with a 1" to "ending with a -1".

- p. 36, line 1–. Insert $+\cdots$ before =. (The left-hand side is an infinite sum.)

- p. 51, line 9–. Change $Q_i = \Pi_i^{(2)}$ to "when Q_i is given by Example 5.5.2(d) for $r = 2$".

- p. 59, line 8. Change "effect" to "affect".

- p. 59, line 9. Change "Since the rows" to "Since the columns".

- p. 59, line 13. Change "Because the columns" to "Because the rows".

- p. 62. Example 5.6.12, line 5. Change "modulo n" to "modulo 2^n".

- p. 63, line 12. Change "sequence" to "sequences".

- p. 65, line 8. Change "Theorem" to "Lemma".

- p. 72, Exercise 5.2(a). Relabel the first part (iii) as part (ii).

- p. 74, Exercise 5.8(a). The stated formula for $T(n,k)$ fails for $n = 0$. Also, it makes more sense to define $T(0,0) = 1$.

- p. 81, Exercise 5.24(d). A solution was found by the Cambridge Combinatorics and Coffee Club (February 2000).

- p. 83, line 1–. Change diagraph to digraph.

- p. 87, equation (5.111). We need to add the further condition that $p_n(0) = \delta_{0n}$. Otherwise, for instance, the polynomials $p_n(x) = (1+x)^n$ satisfy (iv) with $Q = \frac{d}{dx}$ but fail to satisfy (i)–(iii).

- p. 97, Exercise 5.51. It is not true that (ii) implies (i), e.g., when $C(x) = c$. One needs to add the hypothesis that $[x]C(x) \neq 0$, so that $(C(x) - c)^{\langle -1 \rangle}$ exists. Substituting $xC(B(x))$ for x in (ii) yields

  $$xC(B(x))/C(A(xC(B(x)))) = x,$$

  so $C(B(x)) = C(A(xC(B(x))))$. Substituting $B(x)^{\langle -1 \rangle}$ for x yields $C(x) = C(A(B(x)^{\langle -1 \rangle}C(x)))$. Subtract c from both sides and apply $(C - c)^{\langle -1 \rangle}$ to get $x = A(B(x)^{\langle -1 \rangle}C(x))$. Applying $A^{\langle -1 \rangle}$ to both sides gives (i). This argument is due to Daniel Giaimo and Amit Khetan and (independently) to Yumi Odama.

- p. 101, line 3. Change $J_0[(2-t)/\sqrt{t-1}]$ to $J_0\left(\sqrt{-t}\,(2-t)/(1-t)\right)$.

- p. 102, Exercise 5.71. It would be better not to specify the degree d of G, since (as stated in the solution) $d = \lambda_1$.

- p. 103, Exercise 5.74(d). Replace the first two sentences with: Show that all vertices have the same outdegree d. (By (c), all vertices then also have indegree d.)

- p. 103, Exercise 5.74(f). For further information, see F. Curtis, J. Drew, C.-K. Li and D. Pragel, *J. Combinatorial Theory (A)* **105** (2004), 35–50, and the references given there.

- p. 108, Exercise 5.7(a), line 7. Change b_{2n} to b_{2n-k}.

- p. 110, Exercise 5.10(c). This result appears P. Erdǒs and P. Turán, *Acta Math. Acad. Sci. Hungar.* **18** (1967), 151–163 (Lemma 1).

- p. 124, Exercise 5.28. A bijective proof based on Prüfer codes is due to the Cambridge Combinatorics and Coffee Club (December 1999).

- p. 124, Exercise 5.29(b). Update the Pitman reference to *J. Combinatorial Theory (A)* **85** (1999), 165–193. Further results on P_n and related posets are given by D. N. Kozlov, *J. Combinatorial Theory (A)* **88** (1999), 112–122.

- p. 134, Exercise 5.41(a), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.

- p. 136, last line of Exercise 5.41(j). A solution different from the one above was given by S. C. Locke, *Amer. Math. Monthly* **106** (1999), 168.

- p. 137, Exercise 5.45, line 1. Change kxy^k to $(k+1)xy^k$.

- p. 137, Exercise 5.45, line 4. Change this equation to

$$y = x + 2xy + 3xy^2 + \cdots = \frac{x}{(1-y)^2}.$$

- p. 139, Exercise 5.47(c), line 7. A direct combinatorial proof was given by M. Bousquet-Mélou and G. Schaeffer, *Advances in Applied Math.* **24** (2000), 337–368.

- p. 142, line 1. Change L^{n-1} to L^n.

- p. 143, Exercise 5.50(c), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.

- p. 144, Exercise 5.53. The identity

$$4^n = \sum_{j=0}^{n} 2^{n-j} \binom{n+j}{j} \tag{1}$$

follows immediately from "Banach's match box problem," an account of which appears for instance in W. Feller, *An Introduction to Probability Theory and Its Applications*, vol. 1, second ed., Wiley, New York, 1957 (§5.8). This yield a simple bijective proof of (**??**).

- p. 147, Third Solution. The first two lines should be: Equation (5.530) can be rewritten (after substituting $n+k$ for n)

$$(n+k)[x^n]\frac{1}{k}\left(\frac{F^{\langle -1\rangle}(x)}{x}\right)^k = [x^n]\left(\frac{x}{F(x)}\right)^{n+k}. \tag{5.140}$$

- p. 151, Exercise 5.62(b). David Callan observed (private communication) that there is a very simple combinatorial proof. Any matrix of the type being enumerated can be written *uniquely* in the form $P+2Q$, where P and Q are permutation matrices. Conversely $P+2Q$ is always of the type being enumerated, whence $f_3(n) = n!^2$.

- p. 162, lines 13– to 12–. Change "Thus any algebraic power series, as defined in Definition 6.1.1" to "Thus any algebraic function, i.e, any solution η to (6.2)".

- p. 169, item (vi). When there is a region with only two edges, then the neighboring regions will not be convex (as shown in Figure 6.1). Hence when there is a region with two edges the phrase "each a convex k-gon" should be replaced by "each a k-gon".

- p. 175, line 1. Change $\{9, 11\}$ to $\{9, 14\}$.
- p. 175, line 2. Change x^{11} to x^{14}.
- p. 175, line 4. Change v^{11} to v^{14}.
- p. 175, line 11. Change Theorem 5.4.1 to Theorem 5.4.2.
- p. 175, line 2–. Change $k \in K$ to $k \in \mathbb{Z}$.
- p. 176, line 16. Change intesect to intersect.
- p. 176, line 4–. Change $(n+2)$-gon to $(n+1)$-gon.
- p. 192, line 9–. Change $u(0) = 0$ to $v(0) = 0$.
- p. 192, lines 8– to 7–. The example $v = \log(1+x^2) - 1$ is confusing since $v(0) \neq 0$. Nevertheless the series $u(v(x)) = \sqrt{\log(1+x^2)}$ is well-defined formally since we can write

  $$\sqrt{\log(1+x^2)} = x\sqrt{\frac{\log(1+x^2)}{x^2}}.$$

  It would have been better to define

  $$v(x) = \frac{log(1+x)}{x} - 1.$$

  The same remarks apply to Exercise 6.59.
- p. 212, line 1. The statement that Catalan number enumeration originated with Segner and Euler in 1760 (or actually 1758/59 in the cited references) is inaccurate. The enumeration of polygon dissections was stated by Euler in a letter to Goldbach in 1751. This letter is printed in P.-H. Fuss, *Correspondance Mathématique et Physique*, Tome. 1, Acad. Sci. St. Petersburg, 1843; reprinted in The Sources of Science, No. 35, Johnson Reprint Corporation, New York and London, 1968, pp. 549–552.
- p. 212. For further details on the history of Catalan numbers, see P. J. Larcombe and P. D. C. Wilson, *Mathematics Today* **34** (1998), 114–117; P. J. Larcombe, *Mathematics Today* **35** (1999), 25, 89; P. J. Larcombe, *Math. Spectrum* **32** (1999/2000), 5–7; and P. J. Larcombe and P. D. C. Wilson, *Congr. Numerantium* **149** (2001), 97–108.

- p. 212, lines 16–17. I have forgotten the source for the statement that Netto was the first to use the term "Catalan number." Can anyone provide a reference?

- p. 213, line 5–. Change "to Comtet **[19]**" "to Abel [continue??] see Ouvres, vol II, p. 287, point D

- p. 217, Exercise 6.2(a). It needs to be assumed that $F(0) = 0$; otherwise e.g. $F(x) = 1/2$ is a trivial counterexample.

- p. 219, Exercise 6.16. A combinatorial proof was first given by R. Sulanke, *Electronic J. Combinatorics* **7**(1), R40, 2000. A sharper result was subsequently proved combinatorially by D. Callan, A uniformly distributed parameter on a class of lattice paths, math.CO/0310461.

- p. 221, Exercise 6.19(j).This problem appeared as Problem A5 on the 2003 William Lowell Putnam Mathematical Competition. Many participants found the following bijection with 6.19(i) (Dyck paths from $(0, 0)$ to $(2n, 0)$): Let D be a Dyck path from $(0, 0)$ to $(2n, 0)$. If D has no maximal sequence of $(1, -1)$ steps of even length ending on the x-axis, then just prepend the steps $(1, 1)$ and $(1, -1)$ to the beginning of D. Otherwise let R be the rightmost maximal sequence of $(1, -1)$ steps of even length ending on the x-axis. Insert an extra $(1, 1)$ step at the beginning of D and a $(1, -1)$ step after R. This gives the desired bijection.

- p. 224, item **ii**, line 5. Change $S(w) = w$ to $S(w) = 12 \cdots n$.

- p. 228, item **iii**, line 3. To be precise, the displayed sequences should have the intial and final 1's deleted.

- p. 230, Exercise 6.21(b), line 3. Change 5.3.11 to 5.3.12.

- p. 230, Exercise 6.23. Brian Rothbach has pointed out that this problem can be given another stipulation: serieshelpmate in 20. By parity considerations (Black cannot lose an odd number of moves with a knight) the solution is the same as before except Black plays Pa7-a6 instead of Pa7-a5. Thus the number of solutions is $C_{10} = 16796$.

- p. 231, Exercise 6.25(i). This conjecture has been proved by M. Haiman, *J. Amer. Math. Soc.* **14** (2001), 941–1006; `math.AG/0010246`.

- p. 232, Exercise 6.27(c). Robin Chapman has found an elegant argument that there always exists an integral orthonormal basis.

- p. 233, Exercise 6.30, line 3. It would be less ambiguous to change "this exercise" to "that exercise".

- p. 235, Exercise 6.34, line 4. At the end of the line it should be mentioned that the polynomial $g(L_n, q)$ of Exercise 3.71(f) is a further q-analogue of C_n. An additional reference for this polynomial is R. Stanley, *J. Amer. Math. Soc.* **5** (1992), 805–851 (Prop. 8.6).

- p. 236, Exercise 6.34(b), line 8. Change "nonnegative" to "nonpositive".

- p. 238, Exercise 6.38(d), line 1. Change (n, n) to $(n, 0)$.

- p. 239, Exercise 6.39(h). A period is missing at the end of the sentence.

- p. 241, Exercise 6.41, line 1. Change $S^2(w) = w$ to $S^2(w) = 12 \cdots n$.

- p. 246, Exercise 6.55(a), line 4. Change "while $w(t) \geq i + 1$ if t is between $k_i + 1$ and s" to "while $w(t) \geq i + 1$ if $k_i + 1 \leq t \leq s$ or $s \leq t \leq k_i - 1$".

- p. 246, equation (6.62). Change $\sum_{k=1}^{n-1}$ to $\sum_{k=1}^{n}$.

- p. 247, Exercise 6.59. See the item above for p. 192, lines 8– to 7–.

- p. 250, Exercise 6.3, line 3. Replace "$r = s + \frac{1}{2}$ for some $s \in \mathbb{Z}$" with "r cannot be a negative integer".

- page 250, Exercise 6.3, line 3. Change "$r = s + \frac{1}{2}$ for some $s \in \mathbb{Z}$" to $-r \notin \mathbb{P}$. A further reference is C. Banderier and M. Drmota, *Combin. Probab. Comput.* **24** (2015), 1–53.

- p. 250, Exercise 6.3, paragraph 3. The earliest proof that $\sum_{n\geq 0} \binom{2n}{n}^t x^n$ isn't algebraic for any $t \in \mathbb{N}$, $t > 1$, appears in the paper P. Flajolet, *Theoretical Computer Science* **49** (1987), 283–309 (page 294). Flajolet shows that if $\sum a_n x^n$ is algebraic and each $a_n \in \mathbb{Q}$, then a_n satisfies an asymptotic formula

$$a_n = \frac{\beta^n\, n^s}{\Gamma(s+1)} \sum_{i=0}^{m} C_i \omega_i^n + O(\beta^n n^t),$$

where $s \in \mathbb{Q}-\{-1,-2,-3,\ldots\}$, $t < s$, β is a positive algebraic number, and the C_i and ω_i are algebraic with $|\omega_i| = 1$. A simple application of Stirling's formula shows that if $a_n = \binom{2n}{n}^t$, then a_n does not have this asymptotic form when $t \in \mathbb{N}$, $t > 1$.

- p. 250, Exercise 6.4. A complete description of a field of generalized power series that forms an algebraic closure of $\mathbb{F}_p[[x]]$ is given by K. S. Kedlaya, *Proc. Amer. Math. Soc.* **129** (2001), 3461–3470.

- p. 253, last two lines. Change "somewhat general more result" to "somewhat more general result".

- p. 257, Exercise 6.19(k). Update the reference to *J. Integer Seq.* **4** (2001), Article 01.1.3; available electronically at

  `http://www.research.att.com/∼njas/sequences/JIS`.

- p. 258, Exercise 6.19(s), line 1. Change a_i to $a_i - 1$.

- p. 260, line 6–. Change $(c_{j_\ell} + j_\ell - 1, n)$ to (n, j_ℓ).

- p. 260, Exercise 6.19(ee), lines 10–12. The statement that the first published proof of the enumeration of 321-avoiding permutations is due to D. G. Rogers is inaccurate. Knuth provided such a proof in 1973 in the reference given in the first paragraph of the solution. Moreover, a bijective proof was found by D. Rotem, *Inf. Proc. Letters* **4** (1975), 58–61.

- pp. 261–262, Exercise 3.19(pp). A further reference on noncrossing partitions is the nice survey article R. Simion, *Discrete Math.* **217** (2000), 367–409.

- p. 264, Exercise 6.19(iii). It should be mentioned that the diagonals of the frieze patterns of Exercise 6.19(mmm) are precisely the sequences $1a_1a_2\cdots a_n1$ of the present exercise.

- p. 265, Exercise 6.19(lll), lines 3– to 2–. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.

- p. 265, Exercise 6.19(mmm). A couple of additional references to frieze patterns are H. S. M. Coxeter, *Acta Arith.* **18** (1971), 297–310, and H. S. M. Coxeter and J. F. Rigby, in *The Lighter Side of Mathematics* (R. K. Guy and R. E. Woodrow, eds.), Mathematical Association of America, Washingon, DC, 1994, pp. 15–27.

- p. 269, line 1–, to p. 270, line 1. The paper of Postnikov and Stanley has appeared in *J. Combinatorial Theory (A)* **91** (2000), 544–597.

- p. 272, end of Exercise 6.33(c). Yet another proof was given by J. H. Przytycki and A. S. Sikora, *J. Combinatorial Theory(A)* **92** (2000), 68–76, math.CO/9811086.

- p. 272, Exercise 6.34, line 7. Change **a** to **e**.

- p. 274, line 2. Change "D. Vanquelin" to "B. Vauquelin".

- p. 275, Exercise 6.40, line 6. Change M. O. J. to W. O. J.

- p. 278, Exercise 6.53, line 3. Change $Q(x) = x - 2$ to $Q(x) = -x - 2$.

- p. 279, Exercise 6.56(c). In the paper N. Alon and E. Friedgut, *J. Combinatorial Theory (A)* **89** (2000), 133–140, it is shown that $A_v(n) < c^{n\gamma^*(n)}$, where $\gamma^*(n)$ is an extremely slow growing function related to the Ackermann hierarchy. The paper is available at

  `http://www.ma.huji.ac.il/~ehudf`.

- p. 281, Exercise 6.60. An elegant proof based on Gröbner bases was given by Chris Hillar, *Proc. Amer. Math. Soc.* **132** (2004), 2693–2701.

- p. 291, line 9–. In general it is not true that $\hat{\Lambda}_R = \hat{\Lambda} \otimes R$; one only has a natural surjection from the former onto the latter. Equality will hold for instance if R is a finite-dimensional $\mathbb{Q}$-vector space.

- p. 282, Exercise 6.63(b), line 2. Change 1847 to 1848.

- p. 292, line 7. Insert "in" after "role".

- p. 293, lines 11-13. Replace ", and such that the ... exist.)" with a period. (The deleted condition automatically holds.)

- p. 295, Figure 7-3. In the expansion of h_{41}, the coefficient of m_{41} should be 2.
- p. 298, line 10–. Change "if follows" to "it follows".
- p. 300, line 8–. Change $\sum$ to $\prod$.
- p. 301, line 7. Change 1.1.9(b) to 1.9(b).
- pp. 314–315, proof of Proposition 7.10.4. Change λ to λ/μ throughout proof.
- p. 315, Figure 7-4. In the expression for s_3 change the second m_{111} to m_3. Similarly, in the expression for s_4 change the second m_{1111} to m_4.
- p. 317, line 12–. Change "clearly impossible" to "clear".
- p. 322, line 2. Interchange $\tilde{P}$ and $\tilde{Q}$.
- p. 326, line 2. Insert a space after "antichains".
- p. 329, line 15–. Change x's to X's.
- p. 336, line 7 (counting the displayed tableau as a single line). Change 7.8.2(b) to 7.8.2(a).
- p. 346, line 3–. Change "forms a border strip" to "forms a border strip or is empty".
- p. 346, line 1–. Change λ^i/λ^{i+1} to λ^{i+1}/λ^i.
- p. 348, line 9. Change $\chi_\lambda(\mu)$ to $\chi^\lambda(\mu)$.
- p. 352, line 2 of proof of Proposition 7.18.1. Change $\displaystyle\sum_\mu z_\lambda^{-1} f(\lambda) p_\mu$ to $\displaystyle\sum_\lambda z_\lambda^{-1} f(\lambda) p_\lambda$.
- p. 354, line 4. Change "in" to "is".
- p. 354, line 5. Change "a integral" to "an integral".
- p. 355, line 4. Add a period after "nonnegative".

- p. 356, line 1. Insert “character of the” before “action”.
- p. 359, line 6. Change the subscript α_S to $\text{co}(S)$.
- p. 364, line 1. Change $e(D(T))$ to $e(\text{co}(D(T)))$.
- p. 370, line 3 of second proof. Change 1.22(d) to 1.23(d).
- p. 370, line 5–. Change the first row of the middle tableau from 43333311 to 4333311.
- p. 374, first diagram. The 1 at the end of the first row should be in boldface.
- p. 377, line 7; p. 378, line 8; page 378, line 10–. Change $\pi \in B(r, c, t)$ to $\pi \subseteq B(r, c, t)$.
- p. 379, line 5–. Insert π after the first “partition”,
- p. 379, line 4–. Change “similary” to “similarly” and change λ^* to π^*.
- p. 381, middle of page. Replace $\begin{matrix} 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} \\ 0 & & \end{matrix}$ with $\begin{matrix} 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & 1 \\ 0 & & \end{matrix}$.
- p. 383, line 9. Change “$D(w) = T'$ and $D(w^{-1}) = T$” to “$D(w) = D(T')$ and $D(w^{-1}) = D(T)$”.
- p. 394, line 8–. Insert # before $\text{Fix}(w)$.
- p. 395, line 10–. Change “Burnside’s theorem” to “Burnside’s lemma”.
- p. 399, line 15. Change “function” to “functions”.
- p. 399, line 7–. For additional information concerning Craige Schensted, see the webpage `http://ea.ea.home.mindspring.com`.
- p. 404, line 7–. Change A2.2 to A2.4.
- p. 404, line 3–. Littlewood first introduced plethysm in his paper “Polynomial concomitants and invariant matrices,” *J. London Math. Soc.* **11** (1936), 49–55 (page 52).
- p. 405, line 1. Change A2.6 to A2.8.

- p. 405, line 6. Change A2.6 to A2.8.
- p. 416, line 7–. Change $u_{i_{t+2}}$ to $u_{j_{t+2}}$.
- p. 418, line 7. Change "subsequences" to "subsequence".
- p. 419, line 16. Change "was" to "is".
- p. 421, line 9–. Insert "a" after "such".
- p. 421, lines 8– to 7–. Change "second statement of Theorem A1.1.4" to "first assertion of Theorem A1.1.6".
- p. 422, line 3. Change A1.1.4 to A1.1.6.
- p. 424, line 11. Delete "by".
- p. 426, line "tableaux in (A1.137)" to "tableau defined by (A1.137)".
- p. 439, line 7. Delete comma after 156.
- p. 439, reference A1.13. An updated version of this paper of van Leeuwen, entitled "The Littlewood-Richardson rule, and related combinatorics," is available at math.CO/9908099.
- p. 442, Theorem A2.4, line 6. change $\alpha : V \to W$ to $\alpha : W \to W'$.
- p. 442, Theorem A2.4, line 7. Change $v \in V$ to $v \in W$.
- p. 442, Theorem A2.4, line 9. Change "Hence" to "Moreover,".
- p. 443, line 11. Change

  $$\mathrm{char}\,\varphi = (x_1 \cdots x_n)^{-1} = (x_1 \cdots x_n)^{-1} s_\emptyset$$

  to

  $$\mathrm{char}\,\varphi = x_1^{-1} + \cdots + x_n^{-1} = (x_1 \cdots x_n)^{-1} s^{1^{n-1}}$$
- p. 444, line 12. Delete "char".
- p. 444, line 11–. Change "given by (A2.156)" to "generated (as a $\mathbb{C}$-algebra) by (A2.156)".
- p. 447, line 3–. Change $s_1(x_1^{\lambda_i})$ to $s_1(x_1^{\lambda_i}, x_2^{\lambda_i}, \ldots)$.

- p. 450, Exercise 7.4, line 2. Change the exponent $n-1-r$ to $n-1+r$.

- p. 451, Exercise 7.13(a). This exercise is stated incorrectly. For instance, $K_{777,6654}=1$, contrary to the statement of the exercise. One way to state the correct result is as follows. Let the parts of λ' be given by
$$\begin{aligned}\lambda'_1=\cdots=\lambda'_{n_1}>\lambda'_{n_1+1}=\cdots=\lambda'_{n_2}>\lambda'_{n_2+1}=\cdots\\ >\lambda'_{n_{k-1}+1}=\cdots=\lambda'_{n_k}>0.\end{aligned}$$
Define $\lambda^{(j)}=(\lambda'_{n_{j-1}+1},\ldots,\lambda'_{n_j})$ (with $n_0=0$), so $\lambda^{(j)}$ is a partition of rectangular shape. Let μ be a partition with $|\mu|=|\lambda|$, and let
$$\mu^{(j)}=(\mu_{n_{j-1}+1},\ldots,\mu_{n_j}).$$
Then $K_{\lambda\mu}=1$ if and only if $\lambda\geq\mu$ (dominance order) and
  (i) $|\lambda^{(j)}|=|\mu^{(j)}|$ and $\lambda^{(j)}\geq\mu^{(j)}$ for all j.
  (ii) For all $1\leq j\leq k$ either $0\leq\mu'_{n_{j-1}+1}-\lambda'_{n_{j-1}+1}\leq 1$ or $0\leq\lambda'_{n_j}-\mu'_{n_j}\leq 1$.

- p. 452, line 6. Change "k times" to "n times".

- p. 452, Exercise 7/16(a), line 5. Change $c_{i-j}+c_{i+j}$ to $c_{i-j}-c_{i+j}$.

- pp. 452–453, Exercise 7.16(b,e). The formulas for $y_i(n)$ and $u_i(n)$ have been extended to $i\leq 6$ by F. Gascon, *Fonctions de Bessel et combinatoire*, Publ. LACIM **28**, Univ. du Québec à Montréal, 2002 (page 75). In particular,
$$y_6(2n)=6(2n)!\sum_{k=0}^{n}\frac{(10n-13k+8)C_{k+1}}{(n-k+2)!\,(n-k)!\,(k+4)!\,k!},$$
where C_{k+1} denotes a Catalan number.

- p. 459, Exercise 7.30(b), line 2. Change $x_i^{d-1}+x_i^{d-2}x_j+x_i^{d-3}x_j^2+\cdots+x_j^{d-2}x_j^{d-1}$ to $x_i^d+x_i^{d-1}x_j+x_i^{d-2}x_j^2+\cdots+x_j^d$.

- p. 459, Exercise 7.30(c), line 4. Change $d-1$ to d.

- p. 460, Exercise 7.37. For further information on expanding a_δ^2 in terms of Schur functions, see

  `http://www.phys.uni.torun.pl/∼bgw/vanex.html`.

- p. 461, Exercise 7.42, line 2. Change $s_{\tilde{\lambda}}(y)$ to $s_{\widetilde{\lambda'}}(y)$.

- p. 466, line 3–. Change $(\lambda_i - 1)!(\lambda_i' - 1)!$ to $(\lambda_i - i)!(\lambda_i' - i)!$.

- p. 467, line 5. Change ${\scriptstyle\mathfrak{S}_n}$ to $\mathfrak{S}_n$.

- p. 467, Exercise 7.55(b). Let $f(n)$ be the number of $\lambda \vdash n$ satisfying (7.177). Then

  $$\begin{aligned}(f(1), f(2), \ldots, f(30)) = (&1,1,1,2,2,7,7,10,10,34,40,53,61,\\ &103,112,143,145,369,458,579,712,938,1127,\\ &1383,1638,2308,2754,3334,3925,5092).\end{aligned}$$

  The problem of finding a formula for $f(n)$ was solved by Arvind Ayyer, Amritanshu Prasad, and Steven Spallone, `arXiv:1604.08837`.

- p. 467, Exercise 7.59. In order for the bijection $\lambda \mapsto (\lambda^0, \lambda^1, \ldots, \lambda^{p-1})$ given in the solution to part (e) (page 517) to be correct, it is necessary to define a specific indexing of the terms of C_λ. Namely, index a term a by c_i if $i = i_1 - i_0$, where i_1 is the number of 1's weakly to the left of a, and i_0 is the number of 0's strictly to the right of a (so if $a = 1$, then this contributes to i_1). The sequence becomes $\cdots c_{-2}c_{-1}c_0c_1c_2\cdots$ as before, so it suffices to define the indexing by letting the first 1 be c_{1-i_0}, where i_0 is the number of 0's following this 1. Equivalently, $\ell(\lambda) = i_0$.

  Example. If $\lambda = (4,3,3,3,1)$, then $C_\lambda = \cdots 0010110001011\cdots$. The first 1 in this sequence is $c_{1-5} = c_{-4}$. On the other hand, if $\lambda = (3,3,3,2,2,1)$, then $C_\lambda = \cdots 0010100100011\cdots$. Now the first 1 is $c_{1-6} = c_{-5}$.

- p. 468, Exercise 7.59(e), line 3. Change Y^k to Y^p.

- p. 469, Exercise 7.61, line 2. Change "0 or 1" to "0 or ±1".

- p. 474, Exercise 7.70, line 3. Under the second summation sign insert a space between "in" and $\mathfrak{S}_n$.

- p. 477, Exercise 7.79(c), line 1. Change "strenghtening" to "strengthening".
- p. 484, equation (7.193). Change $1 \le i \le j \le n$ to $1 \le i < j \le n$.
- p. 484, Exercise 7.101(b). As in (a), the plane partitions being counted have largest part at most m.
- p. 485, line 4. Change SSYT to "reverse SSYT" (i.e., the rows are weakly decreasing and columns strictly decreasing).
- p. 485, line 5. Change $T_{ij} < n - \lambda_i + i$ to $T_{ij} \le n + \mu_i - i$, and change $n = 3$ to $n = 2$.
- p. 485, lines 6 and 8. Change $t_{32/1,3}(q)$ to $t_{32/1,2}(q)$.
- p. 485, line 7. The five displayed tableaux should be rotated 180°.
- p. 485, line 3–. The asymptotic formula for $a(n)$ should be multiplied by a factor of $1/\sqrt{3\pi}$. The factor $1/\sqrt{\pi}$ was included by Wright and omitted here by mistake. The additional factor $1/\sqrt{3}$ was omitted by Wright, though his proof makes it clear that it should appear. See L. Mutafchiev and E. Kamenov, `math.CO/0601253`.
- p. 491, Exercise 7.9, line 1. Insert ε_λ before $a_{\lambda\mu}e_\lambda$.
- p. 492, Exercise 7.11. Change $\binom{j}{\ell(\mu)-1}$ to $\binom{\ell(\mu)-1}{j}$ (three times).
- p. 493, Exercise 7.13(a). For another proof, see A. N. Kirillov, *Europ. J. Combinatorics* **21** (2000), 1047–1055, arXiv:hep-th/9304099 (Prop. 2.2).
- p. 494, line 4. Change 169–172 to 175–177.
- p. 494, Figure 7-20. Change the labels $\mathrm{R_1h6}$, $\mathrm{R_1h5}$, and $\mathrm{R_2h6}$ to $\mathrm{R_1a6}$, $\mathrm{R_1a5}$, and $\mathrm{R_2a6}$, respectively.
- p. 496, equation (7.199). Change $(m_i(\lambda)!)^{-1}$ to $[\prod_i (m_i(\lambda)!)^{-1}]$.
- p. 497. Exercise 7.22(b), line 2. Change the first $\mathfrak{N}_n$ to $\mathfrak{S}_n$.
- p. 498, Exercise 7.22(h), line 7. Update the Fomin and Greene reference to *Discrete Math.* **193** (1998), 179–200.

- p. 500, displayed tableaux near end of Exercise 7.24. The tableaux T_8 and T_9 are missing the element 8 to the right of 3. Also, the $\{3, 10\}$ under T_9 should be under T_{10}.

- p. 500, line 5–. Change (**??**) to (c).

- p. 502, Exercise 7.27, first displayed equation. Change $(n)_m$ to $(n)_{n-m}$.

- p. 504, line 10–. Update the Babson, et al., reference to *Topology* **38** (1999), 271–299.

- p. 505, Exercise 7.32(a). Stembridge's more general result appears in "Computational aspects of root systems, Coxeter groups, and Weyl characters," in *Interactions of Combinatorics and Representation Theory*, MSJ Memoirs **11**, Math. Soc. Japan, Tokyo, 2001, pp. 1–38 (Theorem 7.4).

- p. 514, Exercise 7.47(m), lines 1–3. Update the reference to R. Stanley, *Discrete Math.* **193** (1998), 267–286.

- p. 514, Exercise 7.47(m). The conjecture of Hamidoune was proved (without using symmetric functions) by M. Chudnovsky and P. D. Seymour, *J. Comb. Theory, Ser. B* **97** (2007), 350–357.

- p. 514, Exercise 7.48(b), lines 2–4. Update the reference to R. Simion and R. Stanley, *Discrete Math.* **204** (1999), 369–396.

- p. 514, lines 4– and 3–. Change "*ibid.*, Cor. 7.1.2" to "R. Stanley, *Electron. J. Combinatorics* **3**, R6 (1996), 22 pp., Cor. 1.2".

- p. 514, line 1–, and p. 515, line 1. "*Ibid.*" refers to the reference in the item above, not to the previous reference in the book.

- p. 515, line 3. "the reference" refers to the reference two items above, viz., R. Stanley, *Electron. J. Combinatorics* **3**, R6 (1996), 22 pp.

- p. 515, Exercise 7.48(g). Further generalizations of shuffle posets are considered by P. Hersh, *J. Combinatorial Theory (A)* **97** (2002), 1–26.

- p. 515, Exercise 7.49. Update this reference to C. Lenart, *J. Algebraic Combin.* **11** (2000), 69–78.

- p. 516, line 8. Change $(\lambda_i - 1)!(\lambda'_i - 1)!$ to $(\lambda_i - i)!(\lambda'_i - i)!$.

- p. 516, Exercise 7.54. The following elegant solution is due to Katerina Kalampogia-Evangelinou. Expand s_λ in terms of power sums and set $x_i = q^{i-1}$ (principal specialization). If μ has no even part, then $p_\mu(1, q, q^2, \dots)$ has no pole at $q = -1$. If λ has an even hook length, then by Corollary 7.21.3 $s_\lambda(1, q, q^2, \dots)$ has a pole at $q = -1$, and the proof follows.

- p. 517, Exercise 7.59(e), line 3. Change Y^k to Y^p.

- p. 517, Exercise 7.59(e), line 9. Change $Y_\emptyset$ to $Y_{p,\emptyset}$, and change Y^k to Y^p.

- p. 518, Exercise 7.59(h), line 1. Change $Y_\emptyset$ to $Y_{p,\emptyset}$, and change Y^k to Y^p.

- p. 518, Exercise 7.59(h), line 2. Change Y^k to Y^p (three times).

- p. 518, Exercise 7.59(h), line 3. Change Y^k to Y^p.

- p. 520, line 3–. Change $\sum_{n\geq 0} h_{2n+1}t^{2n+1}$ to $\sum_{n\geq 0}(-1)^n h_{2n+1}t^{2n+1}$

- p. 534, end of Exercise 7.74. For some connections between inner plethysm and graphical enumeration, see L. Travis, Ph.D. thesis, Brandeis University, 1999, math.CO/9811127.

- p. 535, lines 7–10. Replace the sentence "No proof ... are known." with "A bijective proof of the unimodality of $s_\lambda(1, q, \dots, q^n)$ was given by A. N. Kirillov, *C. R. Acad. Sci. Paris*, Sér. I **315** (1992), 497–501."

- p. 537, Exercise 7.78(f), line 6. Change $s_\mu(x)$ to $s_\mu(y)$ and $s_\nu(x)$ to $s_\nu(z)$.

- p. 539, Exercise 7.85. A further reference to the evaluation of $g_{\lambda\mu\nu}$ is M. H. Rosas, The Kronecker product of Schur functions indexed by two-row shapes or hook shapes, math.CO/0001084.

- p. 542, line 10. Update the Babson, et al., reference to *Topology* **38** (1999), 271–299.

- p. 544, lines 4– to 2–. Update the reference to R. Stanley, *Discrete Math.* **193** (1998), 267–286.

- p. 551, Exercise 7.102(b), lines 2– to 1–. The "nice" bijective proof asked for was given by M. Rubey, A nice bijection for a content formula for skew semistandard Young tableaux, math.CO/0011099. The proof is based on jeu de taquin.

- p. 554, last two lines of Exercise 7.107(a). Update reference to *Annals of Combinatorics* **2** (1998), 103–110.

- p. 556, line 3. Change $n \to \infty$ to $x \to \infty$.

- p. 556, line 6. Change $(x-t)^2$ to $(x-t)$.

- p. 556, line 7. Change $n^{1/6}$ to $n^{1/3}$.

- p. 576, line 7. Change work to word.

- p. 580. Replace index entry "traingle-free graph" with "triangle-free graph".

- p. 580. Change "Valquelin, D." to "Vauquelin, B.".