1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & 1 & 0 & 1 \\ 0 & 3 & 3 & (\alpha - 3) \\ 2 & 1 & \alpha & -2 \end{bmatrix}, \qquad \text{and } \mathbf{b} = \begin{bmatrix} \beta \\ \beta^2 \\ 0 \\ -2 \end{bmatrix} \tag{1}$$

What are the conditions on α and β such that the system $\mathbf{Ax} = \mathbf{b}$:

(a) has no solution?

(b) has an unique solution? Find the unique solution (**Hint:** you will need to row reduced the augmented system to echelon form, and then use the theorems seen in class to impose the conditions on α and β.)

(c) has infinite amount of solutions? Write the solution set in parametric form (**Hint:** You may have two equations for β that have to be satisfied simultaneously. Moreover, you may find useful to know that you can factorize $-2 - \beta + \beta^2 = (\beta - 2)(\beta + 1)$)

2. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & \alpha \end{bmatrix}, \qquad \text{and } \mathbf{b} = \begin{bmatrix} 1 \\ \beta \\ 1 \end{bmatrix} \tag{2}$$

What are the conditions on α and β such that the system $\mathbf{Ax} = \mathbf{b}$:

(a) has no solution?

(b) has an unique solution? Find the unique solution (**Hint:** you will need to row reduced the augmented system to echelon form, and then use the theorems seen in class to impose the conditions on α and β.)

(c) has infinite amount of solutions? Write the solution set in parametric form.

3. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} \tag{3}$$

Show that if $\mathbf{Ax} = 0$, has an unique solution, then $a \neq b$, $b \neq c$ and $a \neq c$. I.e. a, b, c are different!

4. Consider the following lines in $\mathbb{R}^3$

$$L1 : \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, t \in \mathbb{R} \qquad \text{and} \qquad L2 : \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, s \in \mathbb{R}. \tag{4}$$

(a) Show that the lines do not intersect.

(b) Write the equation of the plane containing $L1$ and that is parallel to L2. (**Hint:** Think geometrically!)

5. Consider the following lines in $\mathbb{R}^3$

$$L1 : \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ -5 \end{pmatrix}, t \in \mathbb{R} \qquad \text{and} \qquad L2 : \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -5 \end{pmatrix}, s \in \mathbb{R}. \tag{5}$$

Show that the lines intersect at an unique point, find it.

6. For $c \in \mathbb{R}$, we define the matrix $\mathbf{A_c} \in \mathbb{R}^{3\times 3}$ by

$$\mathbf{A_c} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & 0 \\ 3 & c & 2 \end{bmatrix}. \tag{6}$$

(a) For which values of c does the columns of $\mathbf{A_c}$ span $\mathbb{R}^3$

(b) Let $b = (2, -4, 1)^t$, find the solution $\mathbf{A_0}x = b$