Math 3A
Spring 2016
Midterm 2
5/13/2016
Time Limit: 50 Minutes

Name (Print): ____________________

Student ID ____________________

This exam contains 9 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	50	
2	50	
3	50	
Total:	150	

1. For $c, s \in \mathbb{R}$, we define the matrix $\mathbf{A_{c,s}} \in \mathbb{R}^{3\times 3}$ by

$$\mathbf{A_c} = \begin{bmatrix} s-1 & 1-s & s-1 \\ 2 & 2 & c \\ 3c & c & 2c \end{bmatrix}. \tag{1}$$

   (a) (5 points) Compute $det(\mathbf{A_{c,s}})$.

   (b) (5 points) What are the conditions in c and s such that $\mathbf{A_{c,s}}$ is invertible? (**Hint:** in this case you may need conditions in both s and c)

(c) (10 points) Compute $\mathbf{A_{2,2}^{-1}}$ (i.e. when $c = 2$ and $s = 2$; moreover, you may encounter fractions) .

(d) (5 points) Let $\mathbf{b} = (-1, 2, 4)^t$, find the solution of $\mathbf{A_{2,2}x} = \mathbf{b}$

(e) (5 points) Compute $det(\mathbf{A_{c,s}}^2)$.

(f) (5 points) Compute $det(\mathbf{E_k A_{c,s}})$, where

$$\mathbf{E}_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k & 0 & 1 \end{bmatrix}. \tag{2}$$

(g) (5 points) Compute $det(\mathbf{A_{c,s}}^{-1})$. (**Hint**: you can use the multiplicative property of the determinant and the fact that $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}_3$, where $\mathbf{I}_3$ is the 3×3 identity matrix)

(h) (10 points) Let

$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & 0 \\ 6 & 2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}. \tag{3}$$

Compute $\mathbf{B}^{-1}$.(**Hint:** You may want to partition the matrix in blocks, if you do so, you will find that you have already computed the inverse of one of the blocks, and the others are easy to invert)

2. Let S be the subspace given by

$$S = \text{span}\left\langle \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \end{pmatrix} \right\} \right\rangle$$

(a) (10 points) Find a basis of S.

(b) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation such that $Nul(T) = S$ and

$$T \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -1 \end{pmatrix}, \qquad T \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}. \tag{4}$$

(a) (10 points) Find the rank of T.

(b) (15 points) Find a basis of $Col(T)$.

(c) (15 points) Compute the standard matrix of T.

3. Let $h \in \mathbb{R}$; Define

$$\mathbf{B}_h = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7-2h \end{bmatrix}. \tag{5}$$

(a) (15 points) Compute a basis of Col $\mathbf{B}_0$.

(b) (15 points) Compute a basis of Nul $\mathbf{B}_0$.

(c) (10 points) Find the rank of $\mathbf{B}_0$ and the dimension of Nul $\mathbf{B}_0$.

(d) (10 points) For which values of h the rank of $\mathbf{B}_h$ is 3?