Math 3A
Spring 2016
Midterm 1
4/20/2016
Time Limit: 50 Minutes

Name (Print): ______________________

Student ID ______________________

This exam contains 10 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	50	
2	50	
3	50	
Total:	150	

1. Let

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} e \\ f \end{pmatrix}. \tag{1}$$

   (a) (30 points) Suppose that $a \neq 0$, compute the solution of $\mathbf{Ax} = \mathbf{b}$ using row reduction and provide the conditions on a, b, c, d such that your computations are valid. Express the result as a simplified expression. (**Hint:** remember that you can not divide by zero.)

(b) (5 points) If $a = 0$, and $c \neq 0$, is your above computation still valid? How would you modify it? (explain briefly) (**Hint:** remember that you can swap the equations and the result is the same.)

(c) (5 points) If $a = 0$, $c = 0$, but $b \neq 0$, $d \neq 0$, what are the conditions on e and f such that the system $\mathbf{Ax} = \mathbf{b}$ has a solution? Is the solution unique? (**Hint:** remember that $\mathbf{Ax} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ can be written as a linear combination of the columns of $\mathbf{A}$.)

(d) (10 points) Solve the sytem

$$\begin{bmatrix} \sqrt{2} & 3\sqrt{2} \\ 2\sqrt{2} & \sqrt{2} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5\sqrt{2} \\ 5\sqrt{2} \end{pmatrix}. \tag{2}$$

(**Hint:** You may want to use the formula you just deduced.)

2. Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ given by

$$T(\mathbf{x}) = \begin{pmatrix} x_2 - x_1 + \alpha x_2^2 \\ x_1 + x_2 + 2g x_3 \\ x_1 + x_2 + 2x_3 \\ h x_3 + q \end{pmatrix}, \text{ where } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \tag{3}$$

in which, h, g, q and α are real numbers.

(a) (5 points) What are the condition on α and q such that the transformation is linear? Explain briefly.

(b) (10 points) **From now we suppose that** $q = 0$ **and** $\alpha = 0$. Write the standard matrix $\mathbf{A}$ of the transformation T. (**Hint:** remember that $\mathbf{A}(:, i) = T(\mathbf{e}_i)$.)

(c) (5 points) What are the conditions on h and g such that the transformation T is onto $\mathbb{R}^4$? Explain briefly.

(d) (10 points) What are the conditions on h and g such that the transformation T is one-to-one? Explain briefly.

(e) (10 points) **Suppose that** $h = 0$, then

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 2g \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and let } \mathbf{b} = \begin{pmatrix} 0 \\ 2 \\ r \\ 0 \end{pmatrix}. \tag{4}$$

What are the conditions on r and g such that the system $\mathbf{Ax} = \mathbf{b}$ has a solution? When is the solution unique? (**Hint:** remember to check that the system is consistent, and check the different cases for r and g.)

(f) (10 points) **Suppose that** $h = 0$**,** $g = 1$**, and** $r = 2$. Solve $\mathbf{Ax} = \mathbf{b}$ and give the answer in parametric form.

3. Let

$$\mathbf{A} = \begin{bmatrix} 0 & -\alpha & 2 & 0 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 3 \\ -2 & -2 & 4 & 2\alpha \\ 0 & \alpha & -1 & 2\alpha + 1/2 \end{bmatrix}, \qquad \text{and } \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 2+\alpha \\ 2\beta + \alpha - 2 \end{bmatrix} \tag{5}$$

What are the conditions on α and β such that the system $\mathbf{Ax} = \mathbf{b}$:

(a) (10 points) has no solution?

(b) (20 points) has an unique solution? Find the solution. (**Hint:** you will need to row reduced the augmented system to echelon form, and then use the theorems seen in class to impose the conditions on α and β.)

(c) (20 points) has infinite amount of solutions? Find the solution set in parametric form. (**Hint:** You may have one equations for α and one for β that have to be satisfied simultaneously)