1. Find the distance between the spheres

$$x^2 + y^2 + z^2 = 4$$

and

$$x^2 + y^2 + z^2 + 2x + 4y + 6z - 86 = 0$$

(**Hint :** find the location and radius of each sphere, and then use a simple geometrical argument to show that the distance between the spheres is the distance between the centers minus the radious of both spheres)

2. Find the distance from the origin to the line given by

$$x = 2t + 1; \ y = t - 1; \ z = t - 5; \qquad t \in \mathbb{R}.$$

3. Use the properties of the dot and croos product to show that if $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$ then

$$(\mathbf{a} \times \mathbf{b}) \cdot [(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})] = [\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})] \, .$$

4. A curve called the folium of Decartes is defined by the following parametric equations

$$x = \frac{3t}{1+t^3}, \qquad y = \frac{3t^2}{1+t^3}.$$

   (a) Show that if a point (a, b) lies on the curve, then so does (b, a).
   (b) In which point does the curve intersects the line $x = y$.
   (c) Find the points on the curve in which the tanget lines are horizontal or vertical.
   (d) Show that the line $y = -x - 1$ is a slant assymptote
   (e) Sketch the curve
   (f) Show that the points on the curve satisfy

$$x^3 + y^3 = 3xy$$

   (g) Show that the polar equation of the curve is given by

$$r = \frac{\sec\theta \tan\theta}{1 + \tan^3\theta}.$$

5. Find dy/dx and d^2y/dx^2 of the following curves
   (a)

$$x = t + \sin t; y = t - \cos t.$$

   (b)

$$x = 1 + t^2, y = t - t^3.$$

6. Suppose that you have a canon in a 2 dimensional world. The amoun of powder is contant and you want to throw a canon ball as far as possible. The only parameter that you can control is the angle between the cannon and the floor, which we denote by θ.

You have that the motion is described by parametric equations

$$x(t) = vt\sin(\theta) \qquad \text{and} \qquad y(t) = vt\cos(\theta) - \frac{g}{2}t^2.$$

(a) Compute the distance the cannon ball will travel in the x direction until it touches the ground ($y = 0$).

(b) Plot the function that you found and found the θ for which the distance in x is maximum. (**Hint:** you can use the fact that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, and to find the maximum θ, you can take a look at the graph and convince yourself that the maximum will be atteing when the derivative of the distance function is equal to zero.)

7. Let B be a solid box of length L, width W, and height H. Let S be the set of all points that are a distance at most 1 from some point in B. Express the volume of S in terms of L, W, and H.

8. If

$$\mathbf{r}(t) = t^2\mathbf{i} + t\cos(\pi t)\mathbf{j} + \sin(\pi t)\mathbf{k},$$

compute

$$\int_0^1 \mathbf{r}(t)dt.$$

9. For the curve $\mathbf{r}(t) = \langle \sin^3(t), \cos^3(t), \sin^2 t\rangle$, for $t \in [0, \pi/2]$, find:

(a) the unit tangent vector,

(b) the unit normal vector,

(c) the unit binormal vector, and

(d) curvature.

10. A disk or radious 1 is rotating in the counterclockwise direction at a constant angular speed ω. A particle starts at the center of the disk and moves toward the edge along a fixed radius so that it position at time $t \geq 0$, is given by $\mathbf{r}(t) = t\mathbf{R}(t)$, where

$$\mathbf{R}(t) = \cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}.$$

(a) Show that the velocity $\mathbf{v}$ of the particle is

$$\mathbf{v}(t) = \cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j} + t\mathbf{v}_d$$

where $\mathbf{v}_d = \mathbf{R}'(t)$ is the velocity of a point at the edge of the disk.

(b) Show that the acceleration $\mathbf{a}$ of the particle is given by

$$\mathbf{a} = 2\mathbf{v}_d + t\mathbf{a}_d$$

where, $\mathbf{a}_d = \mathbf{R}''(t)$ is the the acceleration of a point on the edge of the disk. In this case, $2\mathbf{v}_d$ corresponds to the Coriolis acceleration.

(c) Compute the Coriolis acceleration of a moving particle tha tmoves on a rotating disk according to the equation

$$\mathbf{r}(t) = e^{-t}\cos(\omega t)\mathbf{i} + e^{-t}\sin(\omega t)\mathbf{j}.$$

11. Let

$$\mathbf{r}(t) = t\mathbf{i} + \cos(\pi t)\mathbf{j} + \sin(\pi t)\mathbf{k}$$

(a) Sketch the curve.

(b) Find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

(c) let $f(t) = t^2$, compute

$$\frac{d}{dt}\left(\mathbf{r} \circ f\right)(t)$$

and

$$\frac{d^2}{dt^2}\left(\mathbf{r} \circ f\right)(t)$$