

Math 2D  
Winter 2017  
Take-home Quiz 2  
3/06/2017  
Time Limit:  $\infty$

Name (Print): \_\_\_\_\_

Student ID \_\_\_\_\_

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This exam contains 5 pages (including this cover page) and 1 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a “theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
Total:	10	

Do not write in the table to the right.

1. We want to show that the maximum value of the function

$$f(x, y) = \frac{(ax + by + c)^2}{x^2 + y^2 + 1} \quad (1)$$

is  $a^2 + b^2 + c^2$

- (a) (2 points) Compute the gradient of  $f(x, y)$

(b) (2 points) Show that the gradient is zero if

$$ax + by + c = 0 \tag{2}$$

or if

$$ay^2 + a - bxy - cx = 0,$$

$$bx^2 + b - axy - cy = 0.$$

Hint: you will need to factorize the expression that you obtained for the partial derivatives.

(c) (2 points) Check that if

$$ax + by + c = 0 \tag{3}$$

then  $f(x, y) = 0$

(d) (1 point) Check that  $x = a/c$  and  $y = b/c$  are solution of

$$ay^2 + a - bxy - cx = 0,$$

$$bx^2 + b - axy - cy = 0.$$

(e) (1 point) Check that  $f(a/c, b/c) = a^2 + b^2 + c^2$