

Figure 1

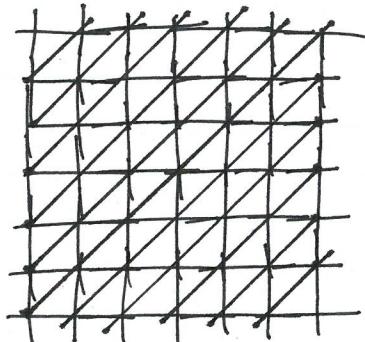
The red lines are  $Z(P)$ , where  $P$  is a minimal degree polynomial vanishing on the six points.

# Figure 2

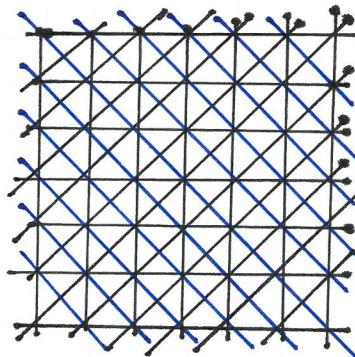
Stars



A grid with many  
3-rich points



A grid with many  
4-rich points



A grid with many  
5-rich points.

(This picture is more  
complicated, so we  
draw it bigger.)

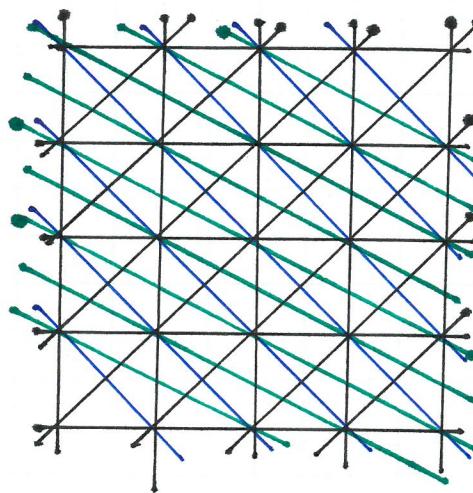
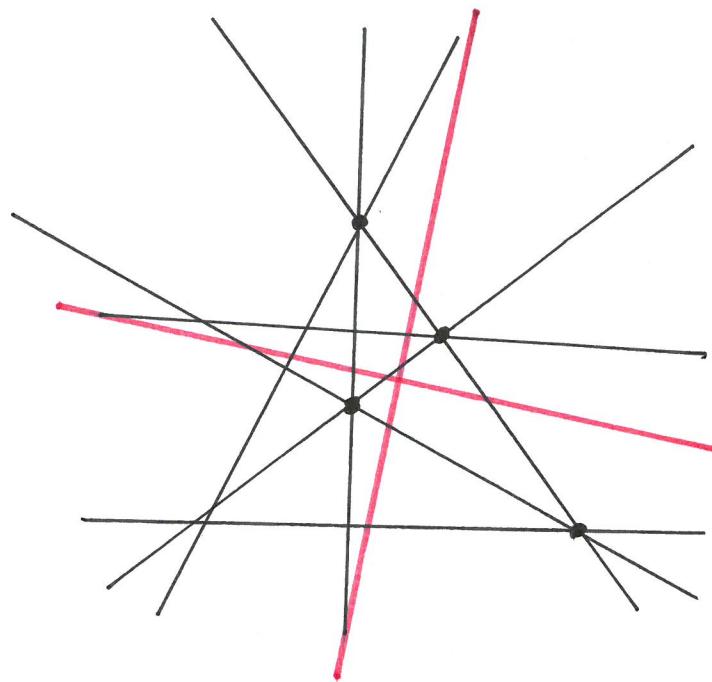


Figure 3.

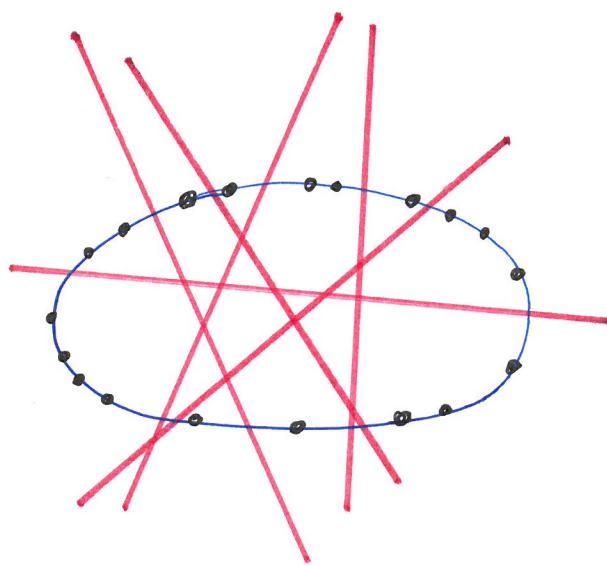


Black lines denote lines of  $\mathcal{L}$ .

Dots denote 3-rich points of  $\mathcal{L}$ .

Here we have  $D=2$  red lines dividing the plane into 4 cells.

Figure 4.



The blue curve is  $\sigma$ . The black dots are points of  $P_r(\mathcal{L})$ . (The lines of  $\mathcal{L}$  are not shown.)

Figure 5.

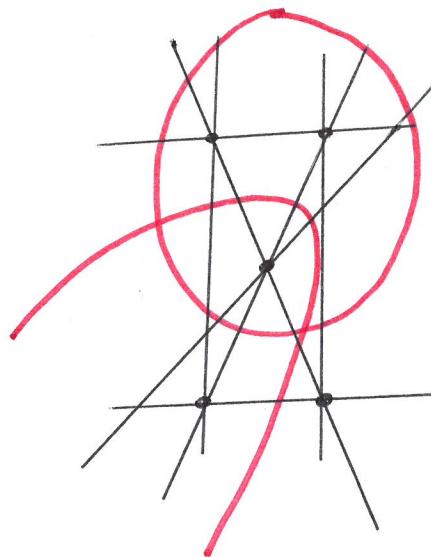
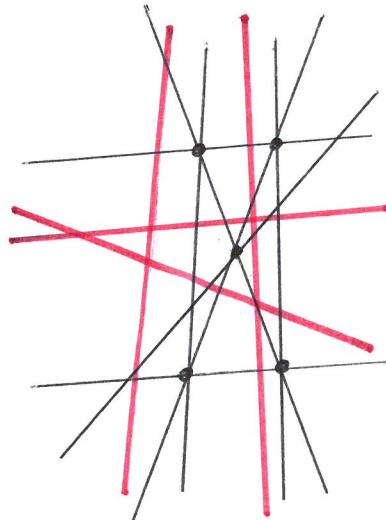
Original cutting

$D$  red lines



Polynomial cutting

degree  $D$  alg. curve,  
 $Z(P)$ , drawn in red.



In the picture  $D=4$ . The curve on the right hand side is a union of two conics, so

$$\text{Deg } P = 2+2 = 4.$$

Choosing  $D$  red lines  
involves  $2D$  real parameters.

Choosing  $P \in \text{Poly}_D(\mathbb{R}^2)$   
involves  $\dim \text{Poly}_D(\mathbb{R}^2) \sim D^2$   
real parameters.

In both cases, a line crosses the red walls at most  $D$  times and so enters at most  $D+1$  cells.