

Brief reading guide for the paper “Sobolev and mean-value inequalities on generalized submanifolds of \mathbb{R}^n ”, Part 1

The Sobolev inequality is in Section 2. The proof is five pages long. Our goal is to study this proof and try to understand the main ideas and look for good ways to explain it. The proof is short, but it’s not so easy to understand ‘what’s going on’ or to explain it to others.

Section 1 is an introduction. There is a page of motivation and then some definitions. Skip example 2 - it’s kind of long and we don’t need it. We will only be looking at Example 3: the example of a submanifold of \mathbb{R}^n . We’ll also never look at Section 3, so the paper we’re studying is effectively under ten pages.

On the first reading, I want you to just try to follow the paper and identify things that are hard to understand. What are the difficulties? What are the problems that the presenter could try to make better? I have in mind that you look at the paper for about an hour - don’t worry if you find it quite hard to follow the paper in that time.

You might want to focus on the special case that the mean curvature is identically zero - in that case we get a simpler inequality: $(\int_M h^{\frac{m}{m-1}})^{\frac{m-1}{m}} \leq C_m \int_M |\nabla h|$. One question to think about is where and how the authors use control of the mean curvature to get at these integrals. How does the mean curvature get into the game? Is there any background knowledge about mean curvature that is necessary or helpful?