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18.433: Combinatorial Optimization
Feb 11th, 2015

Problem set 1

This problem set is due in class on Feb 18th, 2015.

1. Exercise 1-2 of the bipartite matching notes.
2. Exercise 1-4 of the bipartite matching notes.
3. Exercise 1-5 of the bipartite matching notes.
4. (More difficult.) Let $S = \{1, 2, \cdots, n\}$. Let A_k be the set of all subsets of S of cardinality k (thus $|A_k| = \binom{n}{k}$). Let $k < \frac{n}{2}$. Consider the graph G_k with bipartition A_k and A_{k+1}, and with $E = \{(a, b) | a \in A_k, b \in A_{k+1} \text{ and } a \subset b\}$.
   (a) Prove that the maximum matching in G_k has size A_k (remember $k < n/2$).
   (b) Prove *Sperner's lemma*. The maximum number of subsets of S such that no subset is contained into another is $\binom{n}{\lfloor n/2 \rfloor}$.