Signatures of Hermitian forms and unitary representations

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Outline

Introduction

Example: $SL(2, \mathbb{R})$

Character formulas

Hermitian forms

Character formulas for invariant forms

Computing easy Hermitian KL polynomials

Unitarity algorithm

Inspirational story

How does symmetry inform mathematics?

Example. $\int_{-\pi}^{\pi} \sin^5(t)dt = ?$ Zero!

Generalize: $f = f_{\text{even}} + f_{\text{odd}}$, $\int_{-a}^{a} f_{\text{odd}}(t)dt = 0$.

Example. Evolution of initial temp distn of hot ring

$T(0,\theta) = A + B\cos(m\theta)$?

$T(t,\theta) = A + Be^{-c\cdot m^2 t}\cos(m\theta)$.

Generalize: Fourier series expansion of initial temp...

Example. X compact (arithmetic) locally symmetric manifold of dim 128; $\dim\left(H^{28}(X,\mathbb{C})\right) = ?$.

Eight: same as H^{28} for compact globally symmetric space.

Generalize: $X = \Gamma\backslash G/K$, $H^p(X,\mathbb{C}) = H^p_{\text{cont}}(G, L^2(\Gamma\backslash G))$. Decomp L^2:

$$L^2(\Gamma\backslash G) = \sum_{\pi \text{ irr rep of } G} m_\pi(\Gamma)\mathcal{H}_\pi \qquad (m_\pi = \text{dim of some aut forms})$$

Deduce $H^p(X,\mathbb{C}) = \sum_\pi m_\pi(\Gamma)\cdot H^p_{\text{cont}}(G,\mathcal{H}_\pi)$.

General principal: group G acts on vector space V; decompose V; study pieces separately.

Gelfand's abstract harmonic analysis

Topological grp G acts on X, have questions about X.

Step 1. Attach to X Hilbert space $\mathcal{H}$ (e.g. $L^2(X)$).
Questions about $X \rightsquigarrow$ questions about $\mathcal{H}$.

Step 2. Find finest G-eqvt decomp $\mathcal{H} = \oplus_\alpha \mathcal{H}_\alpha$.
Questions about $\mathcal{H} \rightsquigarrow$ questions about each $\mathcal{H}_\alpha$.

Each $\mathcal{H}_\alpha$ is irreducible unitary representation of G: indecomposable action of G on a Hilbert space.

Step 3. Understand $\widehat{G}_u =$ all irreducible unitary representations of G: unitary dual problem.

Step 4. Answers about irr reps $\rightsquigarrow$ answers about X.

Topic today: **Step 3** for Lie group G.

Mackey theory (normal subgps) $\rightsquigarrow$ case G reductive.

What's a unitary dual look like?

$G(\mathbb{R})$ = real points of complex connected reductive alg G

Problem: find $\widehat{G(\mathbb{R})}_u$ = irr unitary reps of $G(\mathbb{R})$.

Harish-Chandra: $\widehat{G(\mathbb{R})}_u \subset \widehat{G(\mathbb{R})}$ = "all" irr reps.

Unitary reps = "all" reps with pos def invt form.

Example: $G(\mathbb{R})$ compact $\Rightarrow \widehat{G(\mathbb{R})}_u = \widehat{G(\mathbb{R})}$ = discrete set.

Example: $G(\mathbb{R}) = \mathbb{R}$;

$$\widehat{G(\mathbb{R})} = \{\chi_z(t) = e^{zt} \ \ (z \in \mathbb{C})\} \simeq \mathbb{C}$$

$$\widehat{G(\mathbb{R})}_u = \{\chi_{i\xi} \ \ (\xi \in \mathbb{R})\} \simeq i\mathbb{R}$$

Suggests: $\widehat{G(\mathbb{R})}_u$ = real pts of cplx var $\widehat{G(\mathbb{R})}$. Almost...

$\widehat{G(\mathbb{R})}_h$ = reps with invt form: $\widehat{G(\mathbb{R})}_u \subset \widehat{G(\mathbb{R})}_h \subset \widehat{G(\mathbb{R})}$.

Approximately (Knapp): $\widehat{G(\mathbb{R})}$ = cplx alg var, real pts $\widehat{G(\mathbb{R})}_h$; subset $\widehat{G(\mathbb{R})}_u$ cut out by real algebraic ineqs.

Today: conjecture making inequalities computable.

Example: $SL(2,\mathbb{R})$ spherical reps

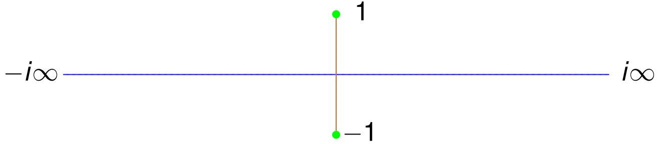
$G(\mathbb{R}) = SL(2,\mathbb{R})$ acts on upper half plane $\mathbb{H}$.

$\rightsquigarrow$ repn $E(\nu)$ on $\nu^2 - 1$ eigenspace of Laplacian $\Delta_{\mathbb{H}}$

$\nu \in \mathbb{C}$ parametrizes line bdle on circle where bdry values live.

Most $E(\nu)$ irr; always unique irr subrep $J(\nu) \subset E(\nu)$.

Spherical reps for $SL(2,\mathbb{R}) \leftrightsquigarrow \mathbb{C}/\pm 1$



Spectrum of $\Delta_{\mathbb{H}}$ on $L^2(\mathbb{H})$ is $(-\infty, -1]$. Gives unitary reps
unitary principal series $\leftrightsquigarrow \{E(\nu) \mid \nu \in i\mathbb{R}\}$.

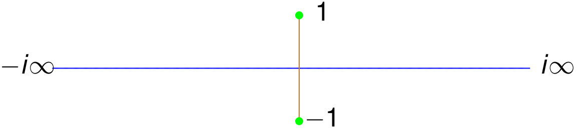
Trivial representations $\leftrightsquigarrow$ [const fns on $\mathbb{H}$] $= J(\pm 1)$.

$J(\nu)$ is Herm. $\Leftrightarrow J(\nu) \simeq J(-\overline{\nu}) \Leftrightarrow \nu \in i\mathbb{R} \cup \mathbb{R}$.
By continuity, signature stays positive from 0 to ± 1.

complementary series reps $\leftrightsquigarrow \{E(t) \mid t \in (-1,1)\}$.

The moral[s] of the picture

Spherical unitary dual for $SL(2,\mathbb{R}) \leftrightsquigarrow \mathbb{C}/\pm 1$



$SL(2,\mathbb{R})$	$G(\mathbb{R})$
$E(\nu), \nu \in \mathbb{C}$	$I(\nu), \nu \in \mathfrak{a}^*_{\mathbb{C}}$
$E(\nu), \nu \in i\mathbb{R}$	$I(\nu), \nu \in i\mathfrak{a}^*_{\mathbb{R}}$
$J(\nu) \hookrightarrow E(\nu)$	$I(\nu) \twoheadrightarrow J(\nu)$
$[-1,1]$	polytope in $\mathfrak{a}^*_{\mathbb{R}}$

Will deform Herm forms
unitary axis $i\mathfrak{a}^*_{\mathbb{R}} \rightsquigarrow$
real axis $\mathfrak{a}^*_{\mathbb{R}}$.

Deformed form pos $\rightsquigarrow$
unitary rep.

Reps appear in families, param by ν in cplx vec space $\mathfrak{a}^*$.

Pure imag params $\leftrightsquigarrow$ L^2 harm analysis $\leftrightsquigarrow$ unitary.

Each rep in family has distinguished irr piece $J(\nu)$.

Difficult unitary reps $\leftrightarrow$ deformation in real param

Signatures for $SL(2,\mathbb{R})$

Recall $E(\nu) = (\nu^2 - 1)$-eigenspace of $\Delta_{\mathbb{H}}$.

Need "signature" of Herm form on this inf-diml space.

Harish-Chandra (or Fourier) idea:
use $K = SO(2)$ break into fin-diml subspaces

$$E(\nu)_{2m} = \{f \in E(\nu) \mid \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot f = e^{2im\theta} f\}.$$

$$E(\nu) \supset \sum_m E(\nu)_m, \qquad \text{(dense subspace)}$$

Decomp is orthogonal for any invariant Herm form.

Signature is $+$ or $-$ for each m. For $3 < |\nu| < 5$

$\cdots$	-6	-4	-2	0	$+2$	$+4$	$+6$	$\cdots$
$\cdots$	$+$	$+$	$-$	$+$	$-$	$+$	$+$	$\cdots$

Deforming signatures for $SL(2, \mathbb{R})$

Here's how signatures of the reps $E(\nu)$ change with ν.

$\nu = 0$, $E(0)$ "$\subset$" $L^2(\mathbb{H})$: unitary, signature positive.

$0 < \nu < 1$, $E(\nu)$ irr: signature remains positive.

$\nu = 1$, form finite pos on $J(1)$ $\leftrightsquigarrow$ $SO(2)$ rep 0.

$\nu = 1$, form has pole, pos residue on $E(1)/J(1)$.

$1 < \nu < 3$, across pole at $\nu = 1$, signature changes.

$\nu = 3$, Herm form finite $- + -$ on $J(3)$.

$\nu = 3$, Herm form has pole, neg residue on $E(3)/J(3)$.

$3 < \nu < 5$, across pole at $\nu = 3$, signature changes. ETC.

Conclude: $J(\nu)$ unitary, $\nu \in [0, 1]$; nonunitary, $\nu \in [1, \infty)$.

$\cdots$	-6	-4	-2	0	$+2$	$+4$	$+6$	$\cdots$	$SO(2)$ reps
$\cdots$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$\cdots$	$\nu = 0$
$\cdots$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$\cdots$	$0 < \nu < 1$
$\cdots$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$\cdots$	$\nu = 1$
$\cdots$	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$\cdots$	$1 < \nu < 3$
$\cdots$	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$\cdots$	$\nu = 3$
$\cdots$	$+$	$+$	$-$	$+$	$-$	$+$	$+$	$\cdots$	$3 < \nu < 5$

From $SL(2,\mathbb{R})$ to reductive G

Calculated signatures of invt Herm forms on spherical reps of $SL(2,\mathbb{R})$.

Seek to do "same" for real reductive group. Need...

List of irr reps $=$ ctble union (cplx vec space)/(fin grp).

reps for purely imag points "$\subset$" $L^2(G)$: unitary!

Natural (orth) decomp of any irr (Herm) rep into fin-diml subspaces $\rightsquigarrow$ define signature subspace-by-subspace.

Signature at $\nu + i\tau$ by analytic cont $t\nu + i\tau$, $0 \leq t \leq 1$.

Precisely: start w unitary (pos def) signature at $t = 0$; add contribs of sign changes from zeros/poles of odd order in $0 \leq t \leq 1 \rightsquigarrow$ signature at $t = 1$.

Categories of representations

G cplx reductive alg $\supset G(\mathbb{R})$ real form $\supset K(\mathbb{R})$ max cpt.

Rep theory of $G(\mathbb{R})$ modeled on Verma modules...

$H \subset B \subset G$ maximal torus in Borel subgp,

$\mathfrak{h}^* \leftrightarrow$ highest weight reps

$V(\lambda)$ Verma of hwt $\lambda \in \mathfrak{h}^*$, $L(\lambda)$ irr quot

Put cplxification of $K(\mathbb{R}) = K \subset G$, reductive algebraic.

$(\mathfrak{g}, K)$-mod: cplx rep V of $\mathfrak{g}$, compatible alg rep of K.

Harish-Chandra: irr $(\mathfrak{g}, K)$-mod $\leftrightsquigarrow$ "arb rep of $G(\mathbb{R})$."

X parameter set for irr $(\mathfrak{g}, K)$-mods

$I(x)$ std $(\mathfrak{g}, K)$-mod $\leftrightarrow x \in X$ $J(x)$ irr quot

Set X described by Langlands, Knapp-Zuckerman:
countable union (subspace of $\mathfrak{h}^*$)/(subgroup of W).

Character formulas

Can decompose Verma module into irreducibles

$$V(\lambda) = \sum_{\mu \leq \lambda} m_{\mu,\lambda} L(\mu) \qquad (m_{\mu,\lambda} \in \mathbb{N})$$

or write a formal character for an irreducible

$$L(\lambda) = \sum_{\mu \leq \lambda} M_{\mu,\lambda} V(\mu) \qquad (M_{\mu,\lambda} \in \mathbb{Z})$$

Can decompose standard HC module into irreducibles

$$I(x) = \sum_{y \leq x} m_{y,x} J(y) \qquad (m_{y,x} \in \mathbb{N})$$

or write a formal character for an irreducible

$$J(x) = \sum_{y \leq x} M_{y,x} I(y) \qquad (M_{y,x} \in \mathbb{Z})$$

Matrices m and M upper triang, ones on diag, mutual inverses. Entries are KL polynomials eval at 1:

$$m_{y,x} = Q_{y,x}(1), \quad M_{y,x} = P_{y,x}(1) \quad (Q_{y,x}, P_{y,x} \in \mathbb{N}[q]).$$

Character formulas for $SL(2,\mathbb{R})$

Recall (eigenspace of $\Delta_{\mathbb{H}}$) $= E(\nu) \longleftrightarrow J(\nu)$. Prefer

$$\text{dual of } E(\nu) = I_{ev}(\nu) \twoheadrightarrow J(\nu).$$

Need discrete series $I_{\pm}(n)$ $(n = 1, 2, \ldots)$ char by

$$I_{+}(n)|_{SO(2)} = n+1,\ n+3,\ n+5\cdots$$
$$I_{-}(n)|_{SO(2)} = -n-1,\ -n-3,\ -n-5\cdots$$

Discrete series reps are irr: $I_{\pm}(n) = J_{\pm}(n)$

Decompose principal series

$$I_{ev}(2m+1) = J_{ev}(2m+1) + J_{+}(2m+1) + J_{-}(2m+1).$$

Character formula

$$J_{ev}(2m+1) = I_{ev}(2m+1) - I_{+}(2m+1) - I_{-}(2m+1).$$

$P_{x,y}$	$I_{ev}(2m+1)$	$I_{+}(2m+1)$	$I_{-}(2m+1)$
$I_{ev}(2m+1)$	1	-1	-1
$I_{+}(2m+1)$	0	1	0
$I_{-}(2m+1)$	0	0	1

Defining Herm dual repn(s)

Suppose V is a $(\mathfrak{g}, K)$-module. Write π for repn map.

Recall Hermitian dual of V

$$V^h = \{\xi : V \to \mathbb{C} \text{ additive} \mid \xi(zv) = \overline{z}\xi(v)\}$$

Want to construct functor

$$\text{cplx linear rep } (\pi, V) \rightsquigarrow \text{cplx linear rep } (\pi^h, V^h)$$

using Hermitian transpose map of operators.

REQUIRES twist by conjugate linear automorphism of $\mathfrak{g}$.

Assume $\sigma\colon G \to G$ antiholom aut, $\quad \sigma(K) = K$.

Define $(\mathfrak{g}, K)$-module $\pi^{h,\sigma}$ on V^h,

$$\pi^{h,\sigma}(X)\cdot\xi = [\pi(-\sigma(X))]^h\cdot\xi \qquad (X \in \mathfrak{g}, \xi \in V^h).$$

$$\pi^{h,\sigma}(k)\cdot\xi = [\pi(\sigma(k)^{-1})]^h\cdot\xi \qquad (k \in K, \xi \in V^h).$$

Classically $\sigma_0 \leftrightsquigarrow G(\mathbb{R})$. We use also $\sigma_c \leftrightsquigarrow$ compact form of G

Invariant forms on standard reps

Recall multiplicity formula

$$I(x) = \sum_{y \le x} m_{y,x} J(y) \qquad (m_{y,x} \in \mathbb{N})$$

for standard $(\mathfrak{g}, K)$-mod $I(x)$.

Want parallel formulas for σ-invt Hermitian forms.
Need forms on standard modules.

Form on irr $J(x) \xrightarrow{\text{deformation}}$ Jantzen filt $I^k(x)$ on std,
nondeg forms $\langle , \rangle^k$ on I^k/I^{k+1}.

Details (proved by Beilinson-Bernstein):

$$I(x) = I^0 \supset I^1 \supset I^2 \supset \cdots, \qquad I^0/I^1 = J(x)$$

$$I^k/I^{k+1} \text{ completely reducible}$$

$$[J(y) \colon I^k/I^{k+1}] = \text{coeff of } q^{(\ell(x)-\ell(y)-k)/2} \text{ in KL poly } Q_{y,x}$$

Hence $\langle , \rangle_{I(x)} \stackrel{\text{def}}{=} \sum_k \langle , \rangle^k$, nondeg form on gr $I(x)$.
Restricts to original form on irr $J(x)$.

Virtual Hermitian forms

$$\mathbb{Z} = \text{Groth group of vec spaces.}$$

These are mults of irr reps in virtual reps.

$$\mathbb{Z}[X] = \text{Groth grp of finite length reps.}$$

For invariant forms…

$$\mathbb{W} = \mathbb{Z} \oplus \mathbb{Z} = \text{Groth grp of fin diml forms.}$$

Ring structure

$$(p, q)(p', q') = (pp' + qq', pq' + q'p).$$

Mult of irr-with-forms in virtual-with-forms is in $\mathbb{W}$:

$$\mathbb{W}[X] \approx \text{Groth grp of fin lgth reps with invt forms.}$$

Two problems: invt form $\langle , \rangle_J$ may not exist for irr J; and $\langle , \rangle_J$ may not be preferable to $-\langle , \rangle_J$.

What's a Jantzen filtration?

V cplx, $\langle,\rangle_t$ Herm forms analytic in t, generically nondeg.

$$V = V^0(t) \supset V^1(t) = \mathrm{Rad}(\langle,\rangle_t), \quad J(t) = V^0(t)/V^1(t)$$

$$(p^0(t), q^0(t)) = \text{signature of } \langle,\rangle_t \text{ on } J(t).$$

Question: how does $(p^0(t), q^0(t))$ change with t?

First answer: locally constant on open set $V^1(t) = 0$.

Refine answer...define form on $V^1(t)$

$$\langle v, w\rangle^1(t) = \lim_{s\to t} \frac{1}{t-s} < v, w >_s, \qquad V_2(t) = \mathrm{Rad}(\langle,\rangle^1(t))$$

$$(p^1(t), q^1(t)) = \text{signature of } \langle,\rangle^1(t).$$

Continuing gives Jantzen filtration

$$V = V^0(t) \supset V^1(t) \supset V^2(t) \cdots \supset V^{m+1}(t) = 0$$

From $t-\epsilon$ to $t+\epsilon$, signature changes on odd levels:

$$p(t+\epsilon) = p(t-\epsilon) + \sum[-p^{2k+1}(t) + q^{2k+1}(t)].$$

Hermitian KL polynomials: multiplicities

Fix σ-invt Hermitian form $\langle , \rangle_{J(x)}$ on each irr having one; recall Jantzen form $\langle , \rangle^n$ on $I(x)^n / I(x)^{n+1}$.

MODULO problem of irrs with no invt form, write

$$(I^n / I^{n+1}, \langle , \rangle^n) = \sum_{y \leq x} w_{y,x}(n)(J(y), \langle , \rangle_{J(y)}),$$

coeffs $w(n) = (p(n), q(n)) \in \mathbb{W}$; summand means

$$p(n)(J(y), \langle , \rangle_{J(y)}) \oplus q(n)(J(y), -\langle , \rangle_{J(y)})$$

Define Hermitian KL polynomials

$$Q^{\sigma}_{y,x} = \sum_n w_{y,x}(n) q^{(l(x)-l(y)-n)/2} \in \mathbb{W}[q]$$

Eval in $\mathbb{W}$ at $q = 1 \leftrightarrow$ form $\langle , \rangle_{I(x)}$ on std.

Reduction to $\mathbb{Z}[q]$ by $\mathbb{W} \to \mathbb{Z} \leftrightarrow$ KL poly $Q_{y,x}$.

Hermitian KL polynomials: characters

Matrix $Q^\sigma_{y,x}$ is upper tri, 1s on diag: INVERTIBLE.

$$P^\sigma_{x,y} \stackrel{\text{def}}{=} (-1)^{l(x)-l(y)}((x,y) \text{ entry of inverse}) \in \mathbb{W}[q].$$

Definition of $Q^\sigma_{x,y}$ says

$$(\text{gr}\, I(x), \langle,\rangle_{I(x)}) = \sum_{y \leq x} Q^\sigma_{x,y}(1)(J(y), \langle,\rangle_{J(y)});$$

inverting this gives

$$(J(x), \langle,\rangle_{J(x)}) = \sum_{y \leq x} (-1)^{l(x)-l(y)} P^\sigma_{x,y}(1)(\text{gr}\, I(y), \langle,\rangle_{I(y)})$$

Next question: how do you compute $P^\sigma_{x,y}$?

Herm KL polys for σ_c

σ_c = cplx conj for cpt form of G, $\sigma_c(K) = K$.

Plan: study σ_c-invt forms, relate to σ_0-invt forms.

Proposition

Suppose $J(x)$ irr $(\mathfrak{g}, K)$-module, real infl char. Then $J(x)$ has σ_c-invt Herm form $\langle,\rangle^c_{J(x)}$, characterized by

$\langle,\rangle^c_{J(x)}$ is pos def on the lowest K-types of $J(x)$.

Proposition $\Longrightarrow$ Herm KL polys $Q^{\sigma_c}_{x,y}$, $P^{\sigma_c}_{x,y}$ well-def.

Coeffs in $\mathbb{W} = \mathbb{Z} \oplus s\mathbb{Z}$; $s = (0,1) \leftrightsquigarrow$ one-diml neg def form.

Conj: $Q^{\sigma_c}_{x,y}(q) = s^{\frac{\ell_o(x)-\ell_o(y)}{2}} Q_{x,y}(qs), \quad P^{\sigma_c}_{x,y}(q) = s^{\frac{\ell_o(x)-\ell_o(y)}{2}} P_{x,y}(qs)$.

Equiv: if $J(y)$ occurs at level k of Jantzen filt of $I(x)$, then Jantzen form is $(-1)^{(l(x)-l(y)-k)/2}$ times $\langle,\rangle_{J(y)}$.

Conjecture is false... but not seriously so. Need an extra power of s on the right side.

Deforming to $\nu = 0$

Have computable conjectural formula (omitting ℓ_o)

$$(J(x), \langle , \rangle^c_{J(x)}) = \sum_{y \leq x} (-1)^{l(x)-l(y)} P_{x,y}(s)(\mathrm{gr}\, I(y), \langle , \rangle^c_{I(y)})$$

for σ^c-invt forms in terms of forms on stds, same inf char.

Polys $P_{x,y}$ are KL polys, computed by `atlas` software.

Std rep $I = I(\nu)$ deps on cont param ν. Put $I(t) = I(t\nu), \ t \geq 0$.

Apply Jantzen formalism to deform t to 0...

$$\langle , \rangle^c_J = \sum_{I'(0) \text{ std at } \nu' = 0} v_{J,I'} \langle , \rangle^c_{I'(0)} \qquad (v_{J,I'} \in \mathbb{W}).$$

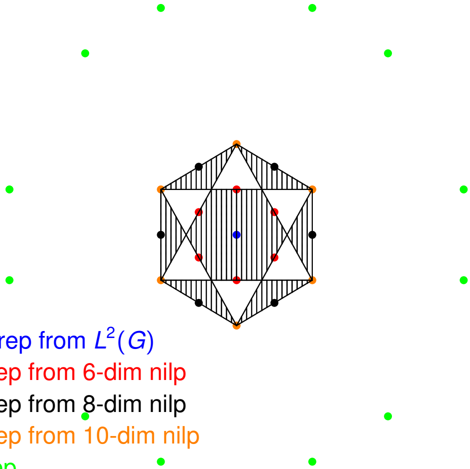
More rep theory gives formula for $G(\mathbb{R})$-invt forms:

$$\langle , \rangle^c_J = \sum_{I'(0) \text{ std at } \nu' = 0} s^{\epsilon(I')} v_{J,I'} \langle , \rangle^0_{I'(0)}.$$

$I'(0)$ unitary, so J unitary $\Leftrightarrow$ all coeffs are $(p, 0) \in \mathbb{W}$.

Example of $G_2(\mathbb{R})$

Real parameters for spherical unitary reps of $G_2(\mathbb{R})$



- Unitary rep from $L^2(G)$
- Arthur rep from 6-dim nilp
- Arthur rep from 8-dim nilp
- Arthur rep from 10-dim nilp
- Trivial rep

Possible unitarity algorithm

Hope to get from these ideas a computer program; enter

- real reductive Lie group $G(\mathbb{R})$
- general representation π

and ask whether π is unitary.

Program would say either

- π has no invariant Hermitian form, or
- π has invt Herm form, indef on reps μ_1, μ_2 of K, or
- π is unitary, or
- I'm sorry Dave, I'm afraid I can't do that.

Answers to finitely many such questions $\rightsquigarrow$ complete description of unitary dual of $G(\mathbb{R})$.

This would be a good thing.

An inspirational story

I was an undergrad at University of Chicago, learning interesting math from interesting mathematicians.

I left Chicago to work on a Ph.D. with Bert Kostant.

After finishing, I came back to Chicago to visit.

I climbed up to Paul Sally's office. Perhaps not all of you know what an interesting mathematician he is.

I told him what I'd done in my thesis; since it was representation theory, I hoped he'd find it interesting.

He responded kindly and gently, with a question:

"What's it tell you about *UNITARY* representations?"

The answer, regrettably, was, "not much."

So I tried again.